Mathematical learning difficulties: Some reflections on the relationship between didactic and a particular kind of psychological research

Difficoltà nell'apprendimento matematico: Alcune riflessioni sulla relazione tra didattica e un tipo particolare di ricerca psicologica

Dificultades en el aprendizaje matemático: Algunas reflexiones sobre la relación entre la didáctica y un tipo particular de investigación psicológica

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Abstract. *The article argues for the avoidance of terms related to learning difficulties in mathematics which, like 'dyscalculia', suggest the presence of a disease or disorder. On the one hand, studies that provide clear evidence that mathematics learning difficulties depend in particular on how children are taught mathematics argue against such a 'medical paradigm'. The article reports on some of the main findings of such studies. On the other hand, as this article will try to show, the term 'dyscalculia' is conceptually deficient, and a certain type of research on dyscalculia is fundamentally flawed in that it does not investigate mathematical learning, but rather its preconditions. Finally, the possible negative consequences of labelling children as dyscalculic and the responsibilities of teachers and the school system are discussed.*

Keywords: learning difficulties; dyscalculia; medical paradigm; labelling.

Sunto. *L'articolo sostiene la necessità di evitare i termini relativi alle difficoltà di apprendimento della matematica che, come 'discalculia', suggeriscono la presenza di una malattia o di un disturbo. Da un lato, gli studi che forniscono prove evidenti del fatto che le difficoltà di apprendimento della matematica dipendono in particolare dal modo in cui i bambini vengono educati alla matematica, sono contrari a questo 'paradigma medico'. L'articolo riporta alcuni dei principali risultati di tali studi. D'altra parte, come questo articolo cercherà di dimostrare, il termine 'discalculia' è concettualmente carente e un certo tipo di ricerca sulla discalculia è fondamentalmente* *difettoso in quanto non indaga l'apprendimento della matematica, ma piuttosto le sue precondizioni. Infine, vengono discusse le possibili conseguenze negative dell'etichettatura dei bambini come discalculici e le responsabilità degli insegnanti e del sistema scolastico.*

Parole chiave: difficoltà di apprendimento; discalculia; paradigma medico; etichettatura.

Resumen. *El artículo aboga por evitar términos relacionados con las dificultades en el aprendizaje de la matemática que, como 'discalculia', sugieren la presencia de una enfermedad o de un trastorno. Por un lado, los estudios que proporcionan pruebas claras de que las dificultades en el aprendizaje de la matemática dependen, en particular, de la forma en cual se educan a los niños en matemática van en contra de este 'paradigma médico'. El artículo informa algunos de los principales resultados de estos estudios. Por otro lado, como este artículo intentará demostrar, el término 'discalculia' es conceptualmente deficiente y cierto tipo de investigación sobre la discalculia es fundamentalmente defectuosa porque no investiga el aprendizaje de la matemática, sino más bien sus condiciones previas. Finalmente, se discuten las posibles consecuencias negativas de etiquetar a los niños como 'discalculicos' y las responsabilidades de los profesores y del sistema escolar.*

Parablas clave: dificultades de aprendizaje; discalculia; paradigma médico; etiquetado.

1. Introduction

The fact that children, and not a few of them, fail early and fundamentally in the area of arithmetic is a phenomenon that not only concerns mathematics education research. It is also a topic in special education, medicine, developmental psychology and, for some years now, very prominently in cognitive and neuropsychology. There are clear differences in the approaches of the disciplines and usually little reference to each other; research tends to be carried out in parallel rather than with combined forces. Kaufmann and Nuerk (2005) speak of "parallel universes" (p. 161).

It is worth noting that this is not a *division of labour* in which maths educators would deal with didactic issues and neuropsychologists, for example, would restrict themselves to research into the neural underpinnings. 'The Number Race' (Wilson & Dehaene, 2004) or 'Calcularis' (von Aster et al., 2014) are just two examples of digital trainings that have their origins in cognitive neuroscience. All in all, mathematics education has not at all a monopoly on the *development of concepts and learning materials* that claim to help overcome learning difficulties in mathematics.

When it comes to *explaining* how and why children develop persistent, massive problems in learning arithmetic, it seems to me that of all the disciplines mentioned above, mathematics education receives the least public attention. In the media, neuropsychologists and cognitive psychologists are more likely to be interviewed and reported on. Parents' associations tend to refer to psychological findings on "dyscalculia" when lobbying school policy on the issue. Politicians, in Italy as elsewhere, refer to psychological approaches to the issue when they decide upon laws and regulations on dealing with learning difficulties at school. This is clear from the fact that such difficulties are grouped together in the relevant regulations under the term 'dyscalculia', with definitions drawn from the psychological literature (e.g., Gazetta ufficiale, 2010; see Gaidoschik, 2022).

As an in-service teacher trainer, I have noticed that many teachers also tend to think about mathematical learning difficulties in terms of what I would like to call in this article – in line with Grissemann (1996) – the 'medical paradigm'. Among teachers (as in psychological research) this view is usually qualified by an admission that teaching matters, too. However, teaching is seen as *secondary*, subordinate: In the medical paradigm the question is how to teach *children with a disorder*, and the disorder is seen as a fact separate from teaching, a precondition that teaching has to deal with.

I consider this paradigm to be theoretically, i.e., in its *explanatory* content, fundamentally flawed, and in its *practical* consequences rather harmful. This is explained in this article.

In order to avoid fruitless misunderstandings, I would like to emphasise at this point: I only want to criticise exactly those positions that I mention and that I characterise with quotations, but not 'psychology' in general, just as I do not justify 'maths education' in general. Certain views are more likely to be found in psychological literature than in didactic literature, but when I speak of the 'medical paradigm' I am referring to a particular view that can be found in the literature of both disciplines, just as there are scholars in both disciplines who see things differently.

2. Some reflections on the word "dyscalculia"

As a first step towards a clearer characterisation of the medical paradigm, a brief reflection on the notion 'dyscalculia' may be useful. The word has two parts, *dys* and *calculia*. The first part, from the Greek, stands for a failure to function; according to wiktionary.org, 'dys-' as prefix 'expresses the idea of difficulty or bad status'. 'Calculia' comes from the Latin 'calculus', as does the English word 'calculate'. So, a literal translation of dyscalculia could be 'difficulty with calculating'. Importantly, however, 'dyscalculia' denotes *a personal characteristic*: someone *has* dyscalculia or even *suffers from* dyscalculia.

Therefore, even if speakers who use this word to refer to certain phenomena do not necessarily mean it in this way, it should be noted: The *linguistic term itself* denotes a widespread phenomenon – quite a number of people have difficulties with arithmetic in a fundamental and persistent way – not in a neutral way, but *linguistically* already containing an explanation or interpretation: The word stands for *something that a person has* or does not have, it denotes a *characteristic* in the sense of a *disorder*. In the words of Baccaglini-Frank and Di Martino (2020): *actions* (a certain, unsuccessful way of dealing with school mathematics) are translated "into properties of the actor", thereby extending "a local, potentially only temporary lack of success into a universal, permanent 'disability'" (Baccaglini-Frank &Di Martino, 2020, p. 545).

This, then, in its briefest form, is the medical paradigm of which this paper is a critique: difficulties in arithmetic are seen as *manifestations (symptoms) of a disorder* that is *inherent in the person*.

The counterargument that this is *not* the case, at least not for all children classified as having 'dyscalculia', will be strengthened in the next section (3) on the basis of a number of *empirical findings* from maths education research.

In the fourth section, I attempt to characterise the fundamental theoretical shortcoming of the medical paradigm more generally, at the *conceptual level*.

Finally, I discuss what I consider to be the sometimes rather harmful *practical consequences* of this view.

3. Mathematic learning difficulties (MLD) as a consequence (also) of mathematic instruction

I begin with some studies that do not examine 'dyscalculia' in general, but rather focus on individual phenomena that form the *core* of persistent learning difficulties in primary school mathematics.

There is widespread agreement in the literature on mathematics education about this core (see, e.g., Gaidoschik et al., 2021; Scherer et al., 2016) as well as there is broad consensus about the fact that learning difficulties in mathematics are not limited to it, and that areas beyond it deserve more attention than they have received so far (see, e.g., Baccaglini-Frank & Di Martino, 2020; Lewis &Fisher, 2016; Verschaffel et al., 2018).

For the purposes of this paper, however, it seems useful to focus on this core. Where psychological literature refers to the content of primary school mathematics, these core-phenomena are also mentioned, but often in a way that is typical of the medical paradigm, in that they are interpreted as 'symptoms' of underlying, more fundamental deficits in the psychometrically measurable 'basic equipment' of these children (e.g., Kucian & von Aster, 2015).

3.1. Three interrelated problem areas typical of MLD

With regard to the core of MLD, at the level of perceptual phenomena, we encounter three major areas of problems that are closely related to each other in quite a number of children (Schipper et al., 2011), that is:

- persistent difficulties in developing alternative ways of solving addition and subtraction problems other than counting strategies;
- persistent difficulties with tasks that require flexible use of the decimal place value system based on conceptual understanding; and
- persistent difficulties with tasks requiring a sound conceptual understanding of basic arithmetic operations, especially multiplication and division (see Gaidoschik et al., 2021, for a more nuanced account).

From the point of view of subject didactics, these problem areas are closely related. I will try to outline these connections briefly.

3.1.1 Basic understanding of numbers

First of all, alternatives to solving addition and subtraction tasks without using counting strategies, apart from recourse to memorised sets of numbers, result primarily from insights into relationships (Gaidoschik, 2019; Sievert et al., 2021). More specifically, children need to understand relationships *within* the number, i.e., the number as a whole reflected in its parts (Björklund et al., 2021; Resnick, 1983; Gerster, 2009), as well as operational relationships, e.g., between an addition and its neighbour addition, or an addition and inverse subtraction (Baroody, 2006; Van de Walle et al., 2023).

On this basis, they are able to 'derive' number facts not yet automated from those they already know by heart. For example, a child who has learnt that eight can be made up of five and three could derive two addition problems (5+3=8, 3+5=8) and two subtraction problems (8−5=3, 8−3=5) only from this one part-whole triple (Fuson, 1983). Knowing 3+3=6 by heart and understanding the relationship between 3+3 and 3+4 ('one more') enables a child to solve also 3+4 without having to count (Sievert et al., 2021); and so on.

With regard to the recall of memorised number sentences ('fact retrieval') as an alternative to calculating by counting, there are clear empirical findings that the automation of basic facts is not *guaranteed* by the insight into relationships (Cumming and Elkins, 1999), but it is considerably *facilitated and promoted* (Gaidoschik, 2012). Conversely, children who know only a few number sentences by heart at the end of the first year of school mostly do not show such insights in relationships, or only sporadically, and in any case do not use them for deriving other facts (see Gaidoschik, 2012).

Fundamental to the development of non-counting strategies is therefore the part-whole understanding of numbers or, as Resnick (1983) formulates ist, the understanding of *numbers as compositions of other numbers*. However, this understanding is by no means self-evident. After all, pre-schooler's dominant access to numbers is through counting. For example, if you count correctly, eight is something that comes *after* five, but it is not necessarily thought of as a composition of five and three (Gaidoschik, 2019).

From a didactic point of view, this points to the urgency of placing a clear focus on part-whole thinking right from the start of formal arithmetic instruction. If children do not take this crucial step in their thinking about and handling of numbers, or if they do not do so early enough, difficulties in further arithmetic lessons are pre-programmed (Gaidoschik et al., 2017a).

3.1.2. The "inner learning hierarchy" of elementary arithmetic

The two other main problem areas that are typical for MLD are, as stated, problems with the decimal place system and insufficient conceptual understanding of arithmetic operations, especially of multiplication and division.

As explained, the three problem areas at the core of MLD are closely related to each other. In a way, *all* of these problems can be understood as extensions of the difficulties of part-whole understanding:

Understanding two-digit and multi-digit numbers in order to be able to work flexibly with them means understanding them as a whole composed of decimally structured parts (Gaidoschik, 2015b; Gerster, 2009).

And when we multiply and divide, we work with units that are (at primary level) larger than one – with fours, nines, sevens and so on. Products need then to be interpreted as a whole composed of such parts in order to understand and flexibly use connections between multiplications or between multiplication and division (Gerster, 2009; Van de Walle et al., 2023).

However, in order to analyse the connections between the problem areas that are at the core of MLD more precisely, we must not remain at such a rather abstract level.

On the one hand, primary school mathematics, with its "learning hierarchy inherent in the nature of the subject" (Wittmann, 2015, p. 199), ensures in a much more concrete way that a child who does not manage to develop non-counting ways to solve addition and subtraction problems, or does so late (measured against the progress of the learning content that he or she is supposed to master in the class), will also stumble in further content already in the first years of school: Having to count with one-digit addition problems necessarily leads to counting with two-digit problems. The larger the numbers, the more complex and error-prone this calculation strategy becomes. Furthermore, children who add and subtract by counting will hardly be able to use derived facts when they need to learn the basic facts of multiplication and will therefore be deprived of a very effective aid to learning these basic facts (Gaidoschik et al., 2017b; Woodward, 2006). Calculating by counting, such as deficits in understanding the decimal system, therefore also have an impact on learning basic multiplication. And this is just *one* example of the aforementioned 'inner hierarchy of learning', which turns out to be a real beast for some children: difficulties in understanding, but also in automating, can hardly be isolated in primary school arithmetic. If there is a problem in one place, there are inevitably many others (Gaidoschik et al., 2021).

This is one side, the cognitive side, of the intertwined deficiencies in understanding basic mathematical content.

The other is the psychological-motivational side: how does a child cope with failure after failure in an important school subject which he or she cannot avoid? This individually different psychological dynamic, which of course also depends on the behaviour of the caregivers, is always involved in learning difficulties, and must also be considered if one wants to understand in any individual case how fundamental deficits in a child's mathematical learning level have built up over the years (Baccaglini-Frank & Di Martino, 2020; Gaidoschik et al., 2021).

3.2. Research on instruction as possible source of learning difficulties

I announced empirical findings that suggest that the explanation for such problems is not only and probably not primarily to be found in the children concerned. In referring to such findings, I will concentrate below on the first and fundamental problem area for everything else: deficits in basic number understanding and the related adherence to calculation by counting.

3.2.1 Indications from research on basic addition and subtraction

I begin with international comparative studies that have repeatedly found statistically significant differences between children from different nations in their reliance on counting to solve simple addition and subtraction problems beyond kindergarten age. Geary et al. (1996) found that in the Chinese classes they studied, the share of counting-based strategies at the end of the first year was 3 per cent, with 91 per cent direct fact retrieval and 6 per cent derived facts. By contrast, in the US classes studied, counting was by far the dominant computational strategy, accounting for 68 per cent at this point. Derived facts were virtually absent in the US classes at both points in the first year of schooling, whereas it was used in more than a third of the tasks in the Chinese classes by the middle of the first year of schooling.

The advantages of Chinese and general East Asian (cf. Sturman, 2015) children (and adults: Campbell & Xue, 2001) over English speakers in arithmetic have repeatedly been confirmed in more recent studies (e.g., Dowker & Li, 2019). They are likely due to a variety of reasons. These range from language influences to parental attitudes towards academic achievement; school drill may also play a role. In any case, as already pointed out by Geary et al. (1996) and a number of other studies (e.g., Fuson & Kwon, 1992), it has to be considered that traditionally in US schools children are rather *encouraged* to use counting for addition and subtraction (cf. Henry & Brown, 2008). Conversely, in the East Asian region, early arithmetic instruction has traditionally emphasised the use of part-whole relations, especially with respect to 10 and 5 (e.g., Zhou & Peverly, 2005).

This is in line with longitudinal and intervention studies, which provide strong evidence that a targeted focus on part-whole relations and derived facts strategies in early arithmetic instruction contributes to children leaving counting strategies behind at an early age (see e.g., Gaidoschik, 2012; Gaidoschik et al., 2017a; Rechtsteiner-Merz, 2013). On the other hand, such a focus on derived facts also benefits children in higher grades who have already developed the habit of calculating by counting. Empirical evidence for this has already been provided by the studies of Thornton (1978; 1990) and Steinberg (1985) in second grade in the USA, and more recently by Koponen et al. (2018).

The latter study explicitly targets children with 'mathematical disabilities' who, over the course of 12 weeks of strategy training, made significant progress in replacing counting strategies with fact retrieval and deduction strategies compared to a control group who received reading support over the same period. Progress was also stable at a 5-month follow-up test (Koponen et al., 2018).

3.2.2 Interim conclusion

What does this mean for the question raised at the beginning of this paper about the relevance of the medical paradigm? One major problem of the children labelled as 'dyscalculic', namely persistent reliance on counting strategies for addition and subtraction, seems to occur more or less frequently depending on the teaching – up to classes in which it is virtually unobservable by the end of the first school year (Gaidoschik et al., 2017a).

This is not to say that good teaching can 100% prevent children from consolidating counting as their main computational strategy. However, with regard to the empirical evidence cited it seems clearly misguided to interpret a child's adherence to counting strategies at higher levels of schooling as a 'symptom' of an inherent disorder, without examining in each individual case what opportunities, stimuli and support the child has had to understand numbers as compositions of numbers and to experience the power of derived facts strategies (Gaidoschik, 2019).

3.2.3. A caveat regarding deficits in the two other core areas

Analogies can be reported, at least to some extent, for the other two main problem areas, that is difficulties with the decimal system and lack of sustainable mental models of the basic operations. I have tried to do this elsewhere (Gaidoschik, 2016), but already there I also admitted that the empirical evidence for instructional influences in these areas is less clear-cut. We certainly need more didactical research on this (Gaidoschik, 2015a).

I therefore formulate with the necessary caution: When we learn, for example, from Moser Opitz (2013) that more than four-fifths of the children or adolescents classified as 'weak in arithmetic' she assessed were overtaxed when calculating, for example, 10,000 minus 100; or from Schäfer (2005) that about half of the fifth graders she interviewed, classified as 'weak in arithmetic', were not able neither to explain the meaning of a given multiplication term using manipulatives nor to invent a suitable word problem; even in such cases of obvious and serious deficits in the area of mathematical foundations we should be careful not to immediately interpret them as an expression of the children's inherent inability. Rather, we should check in each case whether and how, in such cases, the *principle of bundling* and *basic ideas about multiplication* have been developed with this child in class.

And again, what makes me doubt that such difficulties are due to an inherent disorder in some children, are findings from classes in which such difficulties are almost unobservable – classes in which very careful work has been done on understanding the bundling principle (Gaidoschik, 2015a) or on the conceptual understanding of multiplication (Gaidoschik et al., 2017b).

In Italy, research accompanying the PerContare project points in the same direction (Baccaglini-Frank and Bartolini Bussi, 2015). The project aims to prevent learning difficulties in mathematics by providing teachers with didactically sound materials and handouts on the basics of primary school mathematics. Baccaglini-Frank and Bartolini Bussi (2015) report on a study that compared students of classes that participated in the project with a control group of 'traditionally' taught students, and conclude as follows:

> But if one can reduce the number of children testing positive for dyscalculia to such an extent only with careful teaching, one has to wonder what the tests that are used daily for diagnosis really tell us, and more generally what dyscalculia is. (Baccaglini-Frank and Bartolini Bussi, 2015, pp. 108-109; the author's translation)

I agree – and so I continue with some reflections on what dyscalculia is.

4. Dyscalculia: a construct inappropriate for scientific purposes

What if a child should have received arithmetic instruction according to the 'state of art' of mathematics education research on how to prevent MLD, and if the child should also have received appropriate remedial instruction at the first signs of difficulties – and still has fundamental problems with the very basics of the mathematics taught in primary school? May we, should we speak of 'dyscalculia' at least in such cases?

4.1. Lack of content inherent in a negative definition

Here comes a second, more fundamental objection: the term itself is *empty of content*, or more precisely, a *negative definition*. It says that *something is missing* in the child or that the child is not as well developed as they should be. This, however, says nothing about *what is present*: how this child thinks and calculates, however faulty that may be. However, it is precisely this knowledge that we need if we are to help the child overcome his or her misunderstandings and gaps in understanding of basic mathematical content.

For this reason, diagnoses that label a child as 'dyscalculic' are usually of no use when it comes to *supporting* the child. This is also acknowledged by clinical psychologists, e.g., Jacobs and Petermann (2007) in their compendium on the psychological diagnosis of dyscalculia, who state that once a child has gone through all the steps to be diagnosed with dyscalculia, a precise clarification of the content of his or her mathematical development status by a competent educator is still required before any support measures can be planned (see also Gaidoschik, 2022, on this point).

For the same reason, the term 'dyscalculia' seems to me to be unproductive for scientific purposes. With regard to its negative vagueness, it may be compared to, for example, the term 'abdominal pain' in medicine. The generality of 'abdominal pain' may help the lay patient to signal to the attending physician in which direction he should investigate further. But it is too general to be seriously investigated as an object of scientific research into causes, forms of progression, therapeutic options, etc.

I am, of course, aware that *there is* a great deal of research into dyscalculia around the world, and I follow it with professional interest. However, the aforementioned flaw in the starting point – the lack of a clear, *qualitative* definition of what is to be researched – is reflected in the way the research is conducted and in the results of that research.

4.2. The important difference between learning and its prerequisites

On the one hand, the lack of a substantive definition of the category of dyscalculia (see also Lewis & Fisher, 2016; Scherer et al., 2016) means that the sample of children and adolescents studied as ‘dyscalculic’ in relevant research can inevitably only be defined according to ultimately arbitrary quantitative criteria, which differ from study to study. This leads to the often-lamented difficulty of relating the results of different studies to each other.

On the other hand, a certain and not insignificant part of research in neuro- and cognitive psychology on learning difficulties in mathematics does not investigate *mathematical thinking and learning itself* (in order to do this, they would need to identify the learning difficulties with sufficient precision in terms of mathematical content), but rather *its prerequisites*.

And of course, it is important to deal also with the prerequisites if you want to understand why something works or does not work.

However, it seems obvious to me that if we want to understand children’s mathematical thinking and learning, whether they excel or fail in school, we need to analyse *what they think* when they actually *do mathematics*. To do this, of course, we need to look at the *mathematical content*, at the *interactions* in the classroom where most of the mathematical learning takes place, and also at what happens before and after school that can contribute to the success or failure of mathematical learning.

Prerequisites, also on the part of the child – cognitive, motivational, linguistic – should always be kept in mind, too. But a prerequisite must be logically separated from what it is a prerequisite for. As in other areas, it is to be assumed that there are favourable and perhaps even indispensable prerequisites for understanding a certain mathematical content, such as the lack of other prerequisites might be compensated for. In any case, what really matters is what children, with all their prerequisites, actually *do* when they engage in mathematical activities. What is clear from qualitative research on MLD is that, like their more successful peers, children with learning difficulties generalise, compare, abstract, draw conclusions and develop strategies. Their mathematical thinking, knowledge, and skills do not dissolve into prerequisites of any kind. If we are to understand their difficulties, we need to observe them when they do mathematics, give them revealing tasks, analyse how they approach them and what they produce, ask them what they think. Only on this basis can we think about how to help them overcome these difficulties.

Obviously, none of this happens when, for example, children are asked to press a button to indicate which of two groups of dots on the screen is larger, and a magnetic resonance scanner shows which parts of their brain have increased oxygen flow (Kucian & von Aster, 2015). Of course, the neuronal activity imaged in such studies is a prerequisite for thinking, both correct and incorrect thinking: without a brain, there is no calculation. However, the *quality* of a mathematical thought, which determines, for example, whether and with which strategy a child solves a task like 6+7, is not made visible by MRI (magnetic resonance imaging).

Above all, whether a child solves this task by counting or not is obviously not *determined* by his or her brain organ. This is shown by all the children who initially count, but then, with the same brain, learn that it can be done differently – on the basis of *insights* they have gained into numbers and number relationships. Fortunately, you can get at least some indication of what a child was thinking by simply asking them. Of course, there are limits to our efforts to 'enter the child's mind' (Ginsburg, 1997) by interviewing them. Nevertheless, functional MRI is of no help here. It models the *organic substrate* of a child's thinking, but not its *mental content*, not its quality.

Besides functional magnetic resonance imaging, there are many ways to study the preconditions of doing arithmetic, not doing arithmetic itself. In addition to neurobiological and presumed genetic factors, the following factors are discussed as possible influences on the development of dyscalculia in a recent handbook (Landerl et al., 2022): working memory, which in turn is subdivided into central executive, phonological loop, and visuospatial sketchpad; visuospatial processing; general problem-solving ability; verbal skills; attention capacity; and more. To extend this list: Geary et al. (2007) found that "LA [low achievement] children were less fluent in processing numerical information" (p. 1343), so also "speed of processing" should be considered as a "cognitive mechanism underlying achievement deficits in children with mathematics learning disability" (p. 1343). Another candidate as a possible factor for dyscalculia, to which considerable attention and studies have been devoted in recent years, is below average scores on the construct SFON, i.e., spontaneous focussing on numerosity (see Kucian et al., 2012; for a critique, Gaidoschik, 2013). The list could be extended by several more such constructs.

And there is no doubt that in all these areas prerequisites for mathematical learning can be found or at least suspected. Therefore, one may of course investigate whether and how measurable deficits in any of these areas are statistically significantly related to 'dyscalculia' (but always with the shortcoming that, in the absence of clear qualitative criteria, the sample of 'dyscalculics' can only be identified by means of ultimately arbitrary quantitative criteria, which vary from study to study).

And this is what is done in a big number of studies. We certainly learn a great deal about the brain and memory and statistically significant relations between children's performances in different areas from such studies. But for the reasons outlined above, I see only limited use for understanding how children do mathematics, and why some of them find it so difficult, and how to help them. To cite a colleague who comes to the same conclusion:

> Even if new findings [from studies of statistical correlations between MLD and different cognitive prerequisites] on these aspects will become available in the future, this does not yet give an indication of what support measures should be taken. A different kind of research will continue to be needed here – research that looks at concrete mathematical learning processes. (Moser Opitz, 2013, p. 47; the author's translation)

5. Some arguments against labelling children as "dyscalculic"

Let's go back from the criticism of a particular research approach to the criticism of the consequences of the medical paradigm for the practice of dealing with children and adolescents with massive learning difficulties in mathematics. As already explained, the *lege artis* clinical-psychological diagnosis of dyscalculia is not helpful in determining what measures should be taken to overcome or reduce the difficulties; and this is acknowledged by clinical psychologists themselves (Jacobs and Petermann, 2007). What such diagnoses do achieve, however, is to certify that the child has a 'disorder'.

5.1. The double-edged nature of 'disadvantage compensation'

It is important to recognise that, in the circumstances that exist at the moment, some parents may actually want their child to be diagnosed as having dyscalculia. This is understandable if such a diagnosis, as in some nations (e.g., Germany), entitles to financial support for extra-curricular help. Depending on the rules of the national or regional school system, it may also entitle to 'disadvantage compensation' in the form of exemptions for academic assessment and promotion to the next grade or type of school (cf. Gaidoschik et al., 2021).

This, of course, cannot compensate in the long run for the disadvantage of a lack of basic mathematical education. In fact, such "disadvantage compensation" could lead, like it happens in Italy, to a situation where a young adult, despite persisting massive learning difficulties in mathematics, is entitled to study, for example, Primary Education in order to become a teacher of the mathematics that he or she had never the chance to understand.

Rather than "compensating for disadvantage", at least in such cases there is a risk of "reinforcing disadvantage": for the overburdened prospective teacher, but even more so for the children to whom this person might teach mathematics in the future.

5.2. Labelling and inclusion: a contradictory in terms

With regard to *additional targeted support*, i.e., the provision of qualified teachers with sufficient time resources – measures that seem to have real promise in helping children with mathematical learning difficulties (Gaidoschik et al., 2021) – it should be noted that in Italy, for example, a diagnosis of dyscalculia under Law 170 *does not* lead to the provision of additional teaching resources by the school (Baccaglini-Frank & Di Martino, 2020). It does, however, oblige the child's teachers to draw up an Individual Education Plan (IEP), an obligation that often overburdens teachers, who are usually not adequately qualified for this highly demanding task (Gaidoschik, 2022).

Even if teachers *are* qualified, it has to be said that it takes more than a competently designed plan to support children with massive learning difficulties in mathematics: it takes, first and foremost, *additional personal resources* (see above).

Given that such resources do not appear to be available in Italian schools at present, and that there are no targeted efforts on the part of policymakers to change this, the following remark seems to be of little practical significance. From a theoretical point of view, however, it is worth noting:

If the school as a system were to take seriously its role in helping to overcome mathematical learning difficulties, given the uselessness of such a diagnosis for teaching purposes (see above) and the dangers of labelling (see below), it would be highly problematic from a pedagogical point of view to make additional support for children dependent on whether or not they have 'dyscalculia' according to clinical-psychological diagnosis. This would also be in stark contradiction to the principle of inclusive schooling, to which the Italian state in particular has been committed for decades, since efforts to best include a child in mathematics education should not depend on a diagnosis of any kind. It is also for this reason that it remains unclear what such a diagnosis is good for anyway.

5.3. The perils of labelling

But what about the undoubted fact that there are parents of children diagnosed with dyscalculia who are happy that the problem that has often caused them and their child to despair over the years has finally been given a scientific-sounding name and thus an apparent explanation?

First, it has to be stated: In fact, this explains nothing, or rather, the explanation is circular: the child is weak in arithmetic because he has a disorder that weakens them in arithmetic; and that he or she *has* this disorder was essentially extrapolated from the fact that he or she is *weak in a standardised arithmetic test* [and *very* weak, significantly weaker than the age norm; the use of the 'intelligence discrepancy criterion' is no longer recommended for such a diagnosis, at least in the German-speaking world (Deutsche Gesellschaft für Kinder- und Jugendpsychiatrie, Psychosomatik und Psychotherapie e. V. 2018)].

Second: Even if some parents, perhaps also children and adolescents –

including university students – might be happy about the diagnosis of 'dyscalculia', it is important to point out the possible undesirable side effects of such an attribution. In my twenty years or so of working as an out-of-school learning facilitator for children with mathematical learning difficulties, I have met many children who, at least initially, have rejected my efforts to help them learn mathematics by saying that they have dyscalculia, sometimes with the addition that it runs in the family, and that is why they simply cannot learn arithmetic. A diagnosis can be understood by the child as an unchangeable fate and/or taken as an excuse; and there are presumably also other ways how 'mathematics self-concept', in such cases negatively, influences mathematic achievement, as it is shown by empiric research (e.g., Lee & Kung, 2018).

5.4. Why teachers are not to blame but should take responsibility

Unfortunately, I have also come across teachers who have asked me, in my supposed role as an 'expert', to confirm that 'this child has dyscalculia', which for them would have meant that a) they are not to blame and b) they are not responsible.

Now, I am generally in favour of starting from a). I consider it unproductive in any case to skirt around questions of guilt in the face of serious learning difficulties. For even if a closer analysis should reveal failures in teaching, these are usually not the result of unwillingness or even bad intentions. Rather, they result from not knowing better and inadequate teacher education and teacher training. Then many problems are related to school conditions, for which the school system, the policy, is responsible, not the teacher.

So 'blame' is certainly not a constructive category in dealing with learning difficulties. But: A class teacher's *responsibility for the mathematical learning* of all the children in their class should be beyond dispute. Talking about 'dyscalculia' does not help to raise this awareness, at least in my experience. It rather contributes to the attitude of not feeling in charge for 'these children', who would need some sort of 'special treatment', but unfortunately ...

Of course, this also reflects that teachers often feel simply overwhelmed by the task of meeting the needs of children with mathematical learning difficulties. And they *are* objectively overtaxed, even with the highest pedagogical and subject didactic competence, when they teach, for example, in a third class, without additional support in the form of team teaching, and have one or more children in the class who are still calculating by counting, who still know nothing about tens and ones and bundling and unbundling, who may even have learnt the multiplication tables by heart, but have not understood what multiplication is about and therefore cannot apply them.

5.5. The responsibility of school policy

I can understand how, in such circumstances, a teacher might be inclined to think: I am at a loss with the 'dyscalculics'; the parents are responsible, they must go to 'therapy' with the child.

I can understand this kind of thinking. And I think it is all the more important for mathematics educators to criticise this way of some teacher's thinking with good arguments, but at the same time to point out to politicians and school decision-makers that there is a need to invest in school support systems, in initial and in-service training, so that teachers are increasingly able to fulfil the right of all children to receive a basic education in mathematics.

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