Are representations useful in Economic Mathematics? Students' beliefs and self-efficacy beliefs in the case of exponential and logarithmic functions

Le rappresentazioni sono utili in Matematica per l'Economia? Convinzioni e convinzioni di autoefficacia degli studenti nel caso di funzioni esponenziali e logaritmiche

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Abstract. *The study examined the structure of beliefs of the undergraduate students who study economics about the value of mathematics as a tool and the use of representations in understanding exponential and logarithmic functions in relation to their respective self-efficacy beliefs. Results revealed that the students' beliefs and self-efficacy beliefs constitute a coherent model with high loadings. The low interrelation between the students' self-efficacy beliefs and their beliefs indicate that students need positive experiences in order to improve self-efficacy beliefs. The teaching processes in the economic sciences in higher education should reclaim and exploit their positive beliefs about the value of mathematics and the importance of using different modes of representations.*

Keywords: beliefs, self-efficacy beliefs, representations, functions, economic studies.

Sunto. *Lo studio ha preso in esame la struttura delle convinzioni degli studenti universitari che studiano economia sul valore della matematica come strumento e l'uso delle rappresentazioni nella comprensione delle funzioni esponenziali e logaritmiche in relazione alle loro rispettive convinzioni di autoefficacia. I risultati hanno rivelato che le convinzioni e le convinzioni di autoefficacia degli studenti costituiscono un modello coerente con carichi elevati. La bassa interrelazione tra le convinzioni di autoefficacia degli studenti e le loro convinzioni indica che gli studenti hanno bisogno di esperienze positive per migliorare le convinzioni di autoefficacia. I processi di insegnamento delle scienze economiche nell'istruzione superiore dovrebbero rivendicare e sfruttare le loro convinzioni positive sul valore della matematica e sull'importanza di utilizzare diverse modalità di rappresentazione.*

Parole chiave: credenze, credenze di autoefficacia, rappresentazioni, funzioni, studi di economia.

Resumen. *El estudio examinó la estructura de las creencias de los estudiantes de pregrado que estudian economía sobre el valor de la matemática como herramienta y el uso de representaciones en la comprensión de las funciones exponenciales y logarítmicas en relación con sus creencias de autoeficacia. Los resultados revelaron que las creencias y las creencias de autoeficacia de los estudiantes constituyen un modelo coherente con las altas cargas. La baja interrelación entre las creencias de autoeficacia de los estudiantes y sus creencias indica que los estudiantes necesitan experiencias positivas para mejorar las creencias de autoeficacia. Los procesos de enseñanza en las ciencias económicas de la educación superior deben reclamar y explotar sus creencias positivas sobre el valor de la matemática y la importancia de utilizar diferentes modalidades de representaciones.*

Palabras clave: creencias, creencias de autoeficacias, representaciones, funciones, estudios económicos.

1. Introduction

The value of investigating various affective factors in higher education is indicated considering the number of studies (e.g. Crawford, Gordon, Nicholas, & Prosser, 1998; Van Dinther, Dochy, & Segers, 2011) that focus on this field. Recently, researchers (e.g. Dreher, Kuntze, & Lerman, 2015; Stohlmann, Moore, Cramer, & Maiorca, 2015) investigated pre-service teachers' views about using multiple representations in the mathematics classroom. Despite this interest, a limited number of studies have examined the beliefs concerning the use of mathematical representations of undergraduate students enrolled in disciplines other than mathematics or education. It is evident that various fields and disciplines make use of mathematics. For example, mathematical models describe complex phenomena and systems such as stock markets and logistics. Because of all the challenges occurring in the contemporary world, problems of economics and finance, social, environmental, and management sciences are being examined trying to find quantitative models based on mathematical and statistical theories, methods and tools (Melnik, Makarov, & Belair, 2017).

The aim of the study is to examine the structure of important aspects of affective system, the beliefs and self-efficacy beliefs, concerning mathematics as a tool and the use of representations in understanding exponential and logarithmic functions of undergraduate students who study economics. Mathematical representations are increasingly vital as far as expression and communication of ideas in economics is concerned. This is interesting, particularly with respect to the need for public understanding of economics. Furthermore, progressively more activity in the financial market is governed by mathematical models. In fact, economists are using economic and mathematical methods to analyze economic processes and predict any possible outcomes of economic activity (Mandanov & Khasanova, 2014). It should be noted that we concentrate on the concept of function for which a variety of representations such as diagrams, words and symbols are used. Additionally, it is a concept, which is used every day for currency and measurements especially by the economists.

As students of economics are the future professionals in the markets, this study attempts to make both a theoretical and practical contribution to the literature regarding undergraduate students' beliefs and self-efficacy beliefs concerning representations. Considering the fact that beliefs and self-efficacy beliefs are characterized as multidimensional constructs (Bandura, 1989; Goldin, 2002; Buehl & Alexander, 2006), the present study will clarify important aspects of such constructs. Furthermore, a number of studies (e.g. Bartimote-Aufflick, Bridgeman, Walker, Sharma, & Smith 2016; Lizzio, Wilson, & Simons 2002; Chemers, Hu, & Garcia 2001) demonstrate that affective factors reflect the quality of teaching methods in higher education, the teaching environment and the students' outcomes. Thus, the results of studies concerning affective factors such beliefs and self-efficacy beliefs are important to identify corresponding needs for economics in higher education.

2. Literature review

2.1. Beliefs and self-efficacy beliefs

Affective variables are considered by Boekaerts (2003) as important for learning. According to social cognitive theory, human behavior, environment and personal factors (e.g. cognition, emotion and motivation) work accordingly towards each other (Chiu & Klassen, 2010). Beliefs are among the main components of the affective domain (Goldin, 2002). Philipp (2007) defines beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 259). Self-efficacy beliefs are beliefs about the ability to "organize and execute the courses of action required to produce given attainments" (Bandura, 1997, p. 3). According to Bandura's social cognitive theory, self-efficacy acts as agent of motivational orientation that enhances persistence in the case of difficulties, increases intentionality and long-term planning, and determines the development of self-regulation and self-correcting actions in a critical way (Bandura, 2001).

Several researchers (e.g. Gómez-Chacón, Romero-Albaladejo, & Garcia Lopez, 2016) stress the role that beliefs play in the performance of students in mathematics, because beliefs affect the way in which students learn and use mathematics. It was also found by many researchers that self-efficacy influences academic motivation, learning and achievement (Pajares, 1996; Schunk, 1981; Komarraju & Nadler, 2013). Recently, Laging and Voßkamp (2017) include self-efficacy beliefs among the main determinants of mathematics performance of first-year students in several business administration and economics study programs. Therefore, before investigating students' achievement in functions, we conclude that it is important to examine the structure of their beliefs and self-efficacy beliefs.

2.2. The semiotic approach to teaching and learning mathematics

The semiotic approach to teaching and learning mathematics is one of the most important and popular learning theories in mathematics education. Without any doubt one of the most eminent researcher in the domain is Raymond Duval. Some of his books and published articles have contributed to the development of the theoretical background of this domain of research (Duval, 1995, 2017). Moreover, a large number of publications support the theoretical and experimental semiotic approach to learning and teaching mathematics.

Thus, in the current bibliography it is widely accepted that it is important in mathematics teaching and learning to deal with multiple representations (Dreher & Kuntze, 2015). A representation is considered to be a sign or configuration of signs, characters or objects that can demonstrate something else apart from itself (Goldin & Shteingold, 2001). When students recognize the same concept in multiple systems of representations, manipulate the concept within these representations and convert it from one system of representation to another flexibly, engage in deeper sense making and understanding of the particular mathematical concept (Lesh, Post, & Behr, 1987) and see rich relationships (Even, 1998). A very important idea is that a single representation does not describe completely a mathematical construct. Thus, the use of multiple representations for the same mathematical situation is essential to understand mathematics and facilitates students to use effectively the different advantages each representation offers (Duval, 2006). Deliyianni, Gagatsis, Elia and Panaoura (2016) concentrate on representational flexibility and underline the need to give students a variety of situations including representational transformations with varied complexities concerning the same mathematical concept, the systems of representations, and the direction of inter-representational transformations.

Finally a special issue of the journal "Revista Latinoamericana de Investigación en Matemática Educativa" on the topic "Semiotics and Mathematics Education" ("Semiotica y Education Matematica") edited by two famous researchers in the domain of mathematics education, Bruno D'Amore and Louis Radford, gives a very rich spectrum of researches related to semiotic representations and a considerable enforcement in the related theoretical framework.

The large number of researches on the use of representations could be roughly divided in 5 categories:

- The semiotic approach in mathematics education (e.g. Duval, 1995, 2017;

Goldin & Shteingold, 2001; D'Amore & Radford, 2006).

- The role of representational flexibility in understanding and learning mathematics (e.g. Duval, 2006; Gagatsis, Deliyianni, Elia, & Panaoura, 2011, 2017; Deliyianni, Gagatsis, Elia, & Panaoura, 2016).
- The role of representations in problem solving (e.g. Theodoulou, Gagatsis, & Theodoulou, 2004; Gagatsis, & Shiakalli, 2004; Elia, Gagatsis, & Demetriou, 2007; Elia, 2020).
- Cognitive and affect factors related to the use of representations (e.g. Panaoura, Gagatsis, Deliyianni, & Elia, 2009, 2010).
- The definition of a mathematical concept and its relation to the representation flexibility (e.g. Elia, Panaoura, Eracleous, & Gagatsis, 2007; Panaoura, Michael-Chrysanthou, Gagatsis, Elia, & Philippou, 2017).

2.3. The concept of function

Both secondary and higher education should aim to develop "a sense for function" (Eisenberg, 1992, p. 154). In order to achieve this, Eisenberg (1992) suggests that students should have the ability to relate graphical and analytical representations of functions. Because of the nature of the concept of function and of its central role in the mathematics of the secondary education, a substantial number of research studies (e.g. Hitt, 1998; Gagatsis & Shiakalli, 2004; Gagatsis, Elia, & Mousoulides, 2006; Ariza, Llinares, & Valls, 2015; Panaoura, Michael-Chrysanthou, Gagatsis, Elia, & Philippou, 2017) have examined the role that different representations play in the understanding and interpretation of functions. In fact, we have examples of publications in all the above mentioned 5 categories.

On the other hand, Ariza et al. (2015) support that the understanding of economic concepts is related to students' capacity to perform conversions and treatments among the algebraic and graphic representation of the function-derivative relationship when they extract the economic meaning of concavity/convexity in graphs of functions using the second derivative. It is evident that students face difficulties with functions which are caused by the need to connect various function representations such as equations, graphs and tables (Schoenfeld, Smith, & Arcavi, 1993). It is characteristic that a number of students believe that a function must be defined using a single algebraic formula and believe that the table is not a function (Clements, 2001). As Dubinsky and Wilson (2013) emphasize, even students who recognize various types of representations of functions, cannot integrate ideas from various representations. Furthermore, it is important that students perceive the inherent weaknesses of each representation and choose the most appropriate according to the specific context (Nyikahadzoyi, 2015).

Thus, the present study concerns the concept of function and in particular the cognitive and affect factors related to the use of representations in functions that is the fourth category of the above-mentioned classification.

2.4. Hypotheses of the study

Buehl and Alexander (2006) suggest that beliefs concerning knowledge are complex, multidimensional and interactive. As there is much knowledge, there are many beliefs concerning knowledge that are part of one's epistemological belief system. If individuals retain varied and sometimes opposite forms of knowledge in memory, then it is conceivable that the beliefs concerning such knowledge can be similarly varied and even opposite. Thus, while students may profess beliefs concerning the ambiguity of knowledge generally, they may also consider school knowledge to be rather certain (Buehl & Alexander, 2006). We assume that beliefs concerning the use of representations, beliefs about the value of mathematics as a tool in economics and the use of representations in exponential and logarithmic functions constitute distinct components of the domain of beliefs (hypothesis 1). Similarly,

- beliefs concerning the use of representations in general and beliefs concerning the importance of using representations in economics constitute distinct components of beliefs concerning the use of representations (hypothesis 2a);
- beliefs concerning the value of mathematics for an economist and beliefs concerning the value of mathematics for the science of economics constitute distinct components of beliefs concerning the value of mathematics in economics (hypothesis 2b);
- beliefs concerning the use of representations in exponential functions and the use of representations for logarithmic functions constitute distinct components of beliefs concerning the use of representations in functions, respectively (hypothesis 2c).

Mason and Scrivani (2004) indicate that beliefs concerning mathematics, mathematical learning and problem-solving and beliefs concerning the self in relation to mathematics constitute different categories of beliefs. As a result, we assume that self-efficacy beliefs are a distinct component of a higher-order construct that may be thought to stand for a major part of the affective domain regarding representations for the learning of functions (hypothesis 3). Self-efficacy is a multidimensional construct (Bandura, 1989) so self-efficacy beliefs concerning mathematics, the use of representations, the use of representations for exponential functions and the use of representations for logarithmic functions constitute distinct components of the self-efficacy domain (hypothesis 4).

3. Methodology

The study was conducted among the undergraduate students of the Faculty of Economics and Management at the University of Cyprus during the academic year 2015–2016 (all 241 year-1 students, all 130 year-4 students). It is evident that the present study is not a longitudinal one, the 130 students of the fourth year are different from the 241 students of the first year. Despite this, we believe that it is always useful to compare the beliefs of two groups of students of the same Faculty, despite the fact that we cannot give a solid interpretation to the results of the comparison. A questionnaire was compiled in order to investigate the students' beliefs and self-efficacy beliefs concerning mathematics, the use of representations in general and exponential and logarithmic functions in particular. The questionnaire comprises of 56 Likert-type items of five points (one = strongly disagree, five = strongly agree). The reliability of the questionnaire was high (Cronbach's alpha = 0.93). The items were content and face-validated by a professor of economics, an associate professor of management and a professor and an associate professor of mathematics education. The items were piloted before their final administration among 20 year-4 undergraduate students studying in the Faculty of Economics at the Aristotle University of Thessaloniki. Based on the specialists' comments and how the students participating in the pilot phase replied, a number of revisions were made regarding its length and clarity. The items of the questionnaire and their coding may be found in the Appendix.

The questionnaire consisted of statements, which were divided into two main categories. The first category investigated students' beliefs concerning mathematics which included the following dimensions: (a) beliefs concerning the value of mathematics as a tool in relation to their studies, (b) beliefs concerning the value of using different representations, (c) beliefs concerning the value of using representations in relation to their studies, (d) beliefs concerning the value of using representations of exponential and logarithmic functions in relation to their studies. The second category investigated students' self-efficacy beliefs concerning: (a) their mathematics achievement, (b) the use of representations and (c) the use of representations of exponential and logarithmic functions. The questionnaire was administered to the students by the researcher, who is one of the authors, under usual classroom conditions after explaining to them the aim and the significance of the present study. Lecturers left the room, 40 minutes before the end of their teaching period. Students completed the questionnaire in the presence of the researcher who had a passive role as a supervisor.

It should be noted that the questionnaire was anonymous. Despite the fact that the participation was voluntary, all the students filled in the questionnaire. In order to confirm the structure of students' beliefs and self-efficacy beliefs a confirmatory factor analysis model was constructed using Bentler's (1995) structural equation modelling program (EQS). The tenability of a model can be determined using the following measures of goodness of fit: x2/degrees of freedom (df) < 1.95, (comparative fit index) CFI > 0.9 and (root mean square error of approximation) RMSEA < 0.06. Firstly, confirmatory factor analysis was used in order to confirm the structure of the factors of beliefs and self-efficacy beliefs and then it was used in order to examine the interrelations among those affective variables. On the other hand, despite the fact that the two samples of students, do not belong to the same population, it is useful to examine the possible similarity of the beliefs of the two groups of students. That is why, an independent t-test analysis in SPSS statistics was used in order to examine if there were any differences concerning the factors of beliefs and self-efficacy beliefs among year-1 and year-4 students.

4. Results

4.1. Structure of students' beliefs and self-efficacy beliefs

Figure 1 presents the results of the elaborate model that fits the data reasonably well (x2/df = 1.67, df = 1283, CFI = 0.948, RMSEA = 0.043). The third order model that is considered appropriate, in respect to the hypotheses, for interpreting students' beliefs and self-efficacy beliefs involves 10 first-order factors, 4 second-order factors and a third-order factor. The four second-order factors that correspond to students' self-efficacy beliefs concerning mathematics and the use of representations (F11), their beliefs concerning the use of representations (F12), beliefs concerning the value of mathematics in economics and for an economist (F13) and beliefs concerning the use of representations in exponential and logarithmic functions (F14) are highly regressed on a third-order factor that stands for the affective domain regarding the representations for learning the concept of functions. Therefore, hypothesis 1 and 3 stated above are verified.

The four first-order factors (F1-F4) express the students' self-efficacy beliefs. Specifically, first-order factors refer to the following: self-efficacy beliefs concerning mathematics (F1), the use of representations (F2), the use of representations for exponential functions (F3) and the use of representations for logarithmic functions (F4). Those first-order factors regressed on a second-order factor concerning the students' self-efficacy beliefs, as hypothesis 4 supposes.

The second second-order factor expresses students' beliefs concerning the use of representations. It consists of two first-order factors concerning their beliefs concerning the use of representations (F5) and the importance of using representations in economics (F6). The third second-order factor expresses students' beliefs concerning the value of learning mathematics. It consists of two first order factors concerning the value of mathematics in economics (the first one is about the economists – F7 and the second one is about the science of economics – F8). Finally, the fourth second-order factor expresses students' beliefs concerning the use of representations in function concepts. It consists of two first-order factors about students' beliefs concerning the use of representations for the exponential functions (F9) and logarithmic functions (F10). Thus, hypotheses 2a to 2c are confirmed as well.

The high loadings indicate that there is a coherent model as far as undergraduate students' beliefs and self-efficacy beliefs concerning the specific domain are concerned. Students' self-efficacy beliefs regressed with a lower loading (0.526) on the third-order factor than their beliefs concerning the value of the representations in general (0.776), the value of mathematics (0.933) and the use of representations for the exponential and logarithmic functions (0.918). Thus, the results provide evidence that students have stronger beliefs than self-efficacy beliefs.

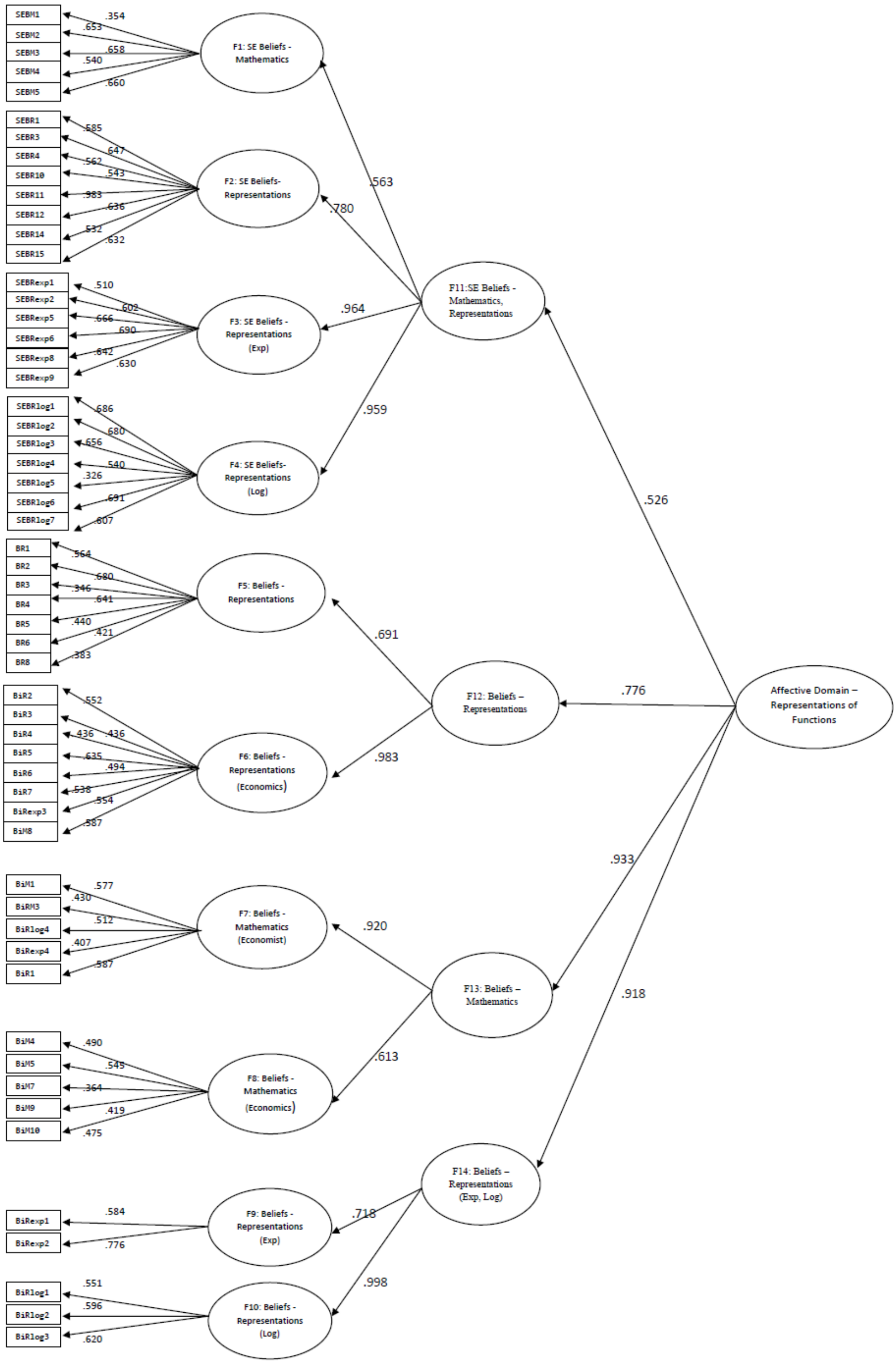

Figure 1. The CFA model of students' beliefs and self-efficacy (SE) beliefs.

4.2. Interrelations among the students' beliefs and self-efficacy beliefs

Figure 2 presents the interrelations among undergraduate students' beliefs and self-efficacy beliefs concerning mathematics and the use of representations for functions. In particular, we concentrated on the interrelations between the second-order factors in order to examine: a) the impact of beliefs on self-efficacy beliefs and b) the impact of more general beliefs concerning the value of mathematics and representations on more specific beliefs concerning the use of representations in logarithmic and exponential functions. The highest statistically significant loading is the one between the students' beliefs concerning the value of representations and their beliefs concerning the value of mathematics (0.918). This finding is explained considering the fact that students relate mathematics to the presence of symbols, graphs and models. At the same time the interrelations between their beliefs concerning the value of mathematics for the economic science is highly related to their beliefs concerning the use of representations for functions (0.873). It seems that students who have strong beliefs concerning the value of mathematics as one of the major subjects in their studies, have at the same time strong beliefs concerning the value of using representations in general and solving exponential and logarithmic functions.

The interrelation between students' beliefs concerning the use of representations in general and exponential and algorithmic functions is close as well (0.759). This is explained by the fact that university students appreciate the value of representations broadly.

The interrelation among students' self-efficacy beliefs concerning mathematics and their beliefs concerning the use of representations in general is statistically significant, but not as high (0.594) as the aforementioned ones. It seems that there are students who have strong beliefs, but they do not have the corresponding strong self-efficacy beliefs. Similarly, the interrelation among students' beliefs concerning the use of representations in the case of exponential and logarithmic functions and their self-efficacy beliefs in mathematics is statistically significant, but not high (0.573). However, the interrelation among students' beliefs concerning the value of mathematics for the economic science with their self-efficacy beliefs concerning mathematics is higher (0.653) than the aforementioned one. In general, it seems that there are students who have strong beliefs but do not have at the same time strong self-efficacy beliefs. A possible explanation for this is that students face difficulties in mathematics that do not allow them to achieve high performance.

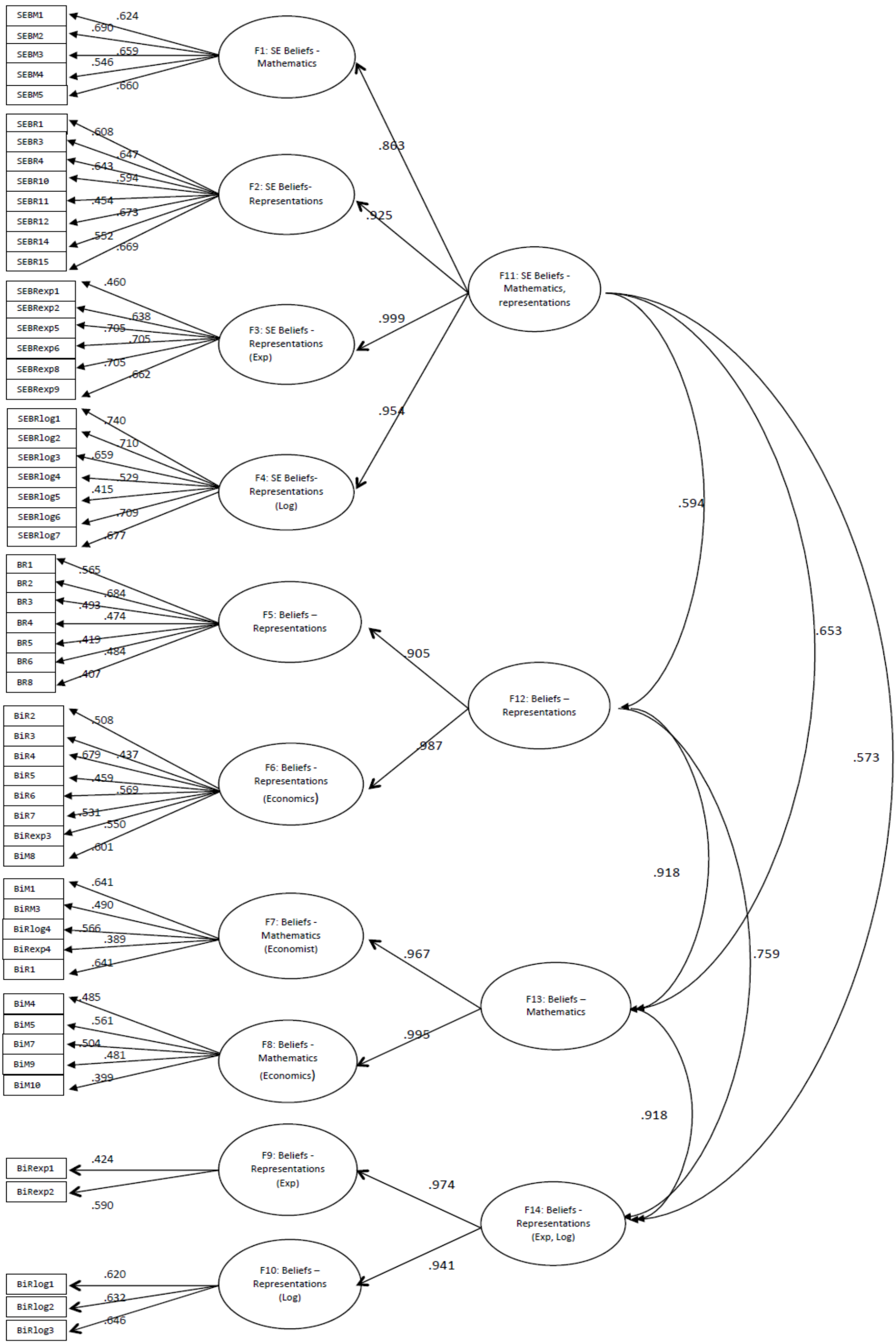

Figure 2. Interrelations among students' beliefs and self-efficacy beliefs.

4.3. Differences between year-1 and year-4 students' beliefs and self-efficacy beliefs

We have already noticed, that the present study is not a longitudinal one, the two samples of students are different. Thus, any comparisons of the means and standard deviations of each affective factors for year-1 and year-4 students, of the following Table 1 must be interpreted with caution.

Table 1
Mean and standard deviations of affective factors in each year

	1st year		4th year	
Factors	$\bar{x}$	S.D.	$\bar{x}$	S.D.
F1: SE Beliefs - Mathematics	3,46	0,71	3,64	0,67
F2: SE Beliefs - Representations	3,47	0,63	3,49	0,66
F3: SE Beliefs - Representations (Exp)	3,30	0,64	3,34	0,78
F4: SE Beliefs - Representations (Log)	3,16	0,65	3,18	0,71
F5: Beliefs - Representations	3,94	0,54	3,91	0,59
F6: Beliefs - Representations (Economics)	3,75	0,57	3,83	0,54
F7: Beliefs - Mathematics (Economist)	3,46	0,65	3,46	0,74
F8: Beliefs - Mathematics (Economics)	3,58	0,69	3,69	0,66
F9: Beliefs - Representations (Exp)	3,00	0,87	2,70	0,90
F10: Beliefs - Representations (Log)	3,24	0,82	3,12	0,84

According to the results, there is no statistical significance difference between year-1 and year-4 students' self-efficacy beliefs concerning mathematics and the use of representations (F1-F4), their beliefs concerning the value of representations (F5-F6), the value of mathematics for the science of economics (F7-F8) and students' beliefs concerning the use of the representations for the logarithmic functions (F10).

However, year-1 students' beliefs concerning the use of representations for the exponential functions (F9) are significantly stronger ($p<0.05$) in relation to those of year-4 students. This could be explained by the fact that the two samples of the students are not the same. We could also take into account the fact that the mathematics modules which students attend in year 1 and their requirements involved mainly exponential functions. Students who study economics at the University of Cyprus take a refresher mathematics module in the first semester. The module introduces mainly the basics of exponential functions. In the second and third semester students are enrolled in two modules in mathematical economics that focus on calculus and optimization. These modules do not offer any additional treatment of exponential functions but typically use them as examples in a limited number of applications. During the rest of their studies students generally do not often come across exponential functions as they are not commonly used to economic problems. Students who take optional modules in finance are more likely to encounter them as they are more commonly used to that field in order to analyze concepts such as compounding and present value calculations.

5. Discussion

Economics is a science that involves a large number and a wide variety of concepts, variables and models (Moosevian, 2016) and uses mathematics in order to explain phenomena and support assumptions. The present study concentrates on students' beliefs concerning the value of mathematics and representations for understanding functions, and their corresponding self-efficacy beliefs during their studies in economics. The confirmatory factor analysis indicated that ten first-order factors were required to account for the second-order factors for the beliefs and the self-efficacy beliefs concerning the role of mathematics in the science of economics, the role of representations in mathematics learning in general and in exponential and logarithmic functions in particular. The factor loadings of the proposed three-order model suggested that the affective domain regarding representations for the learning of functions constituted a multifaceted construct in which general beliefs concerning the use of representations, particular beliefs concerning the use of representations in functions, beliefs concerning the value of mathematics and self-efficacy beliefs concerning their ability in mathematics and the use of representations were involved. It should be mentioned that these constructs do not cover the whole spectrum of the affective domain regarding representations of functions. We suggest, however, that they reveal some needs that students have which if taken into consideration during the organization of economics modules may contribute to the improvement of students' beliefs and self-efficacy beliefs.

It is interesting that year-1 and year-4 students' self-efficacy beliefs concerning mathematics and the use of representations, their beliefs concerning the value of representations, the value of mathematics for the science of economics and students' beliefs concerning the use of representations for the concept of function do not differ significantly. Taking into account that the present study is not a longitudinal one, we cannot give a solid interpretation to the above-mentioned stability of beliefs.

Findings indicate that students of economic sciences have stronger beliefs concerning the value of mathematics for their studies and for the significance of using representations in general and the concept of function in particular than the corresponding self-efficacy beliefs. Although we do not have data about their corresponding mathematics performance, we believe that the difficulties students face during the first year of their studies or their previous experiences during the secondary education may act as obstacles to improve their self-efficacy beliefs. The next phase of our research will examine the interrelation of self-efficacy beliefs with previous mathematics achievements and with recent mathematics experience. A number of previous studies (e.g. Panaoura, Michael-Chrysanthou, Gagatsis, & Philippou, 2017; Sajka, 2003) indicate that students face many difficulties in understanding functions at different ages of secondary education which may affect their beliefs in higher education. Therefore, a possible explanation for our findings is that their experiences at secondary education and in particular their mathematical performance as it is revealed by their final grades in mathematics do not allow some students to develop strong self-efficacy beliefs (Panaoura, Gagatsis, Deliyianni, & Elia, 2010). They need strong and continuous positive experiences in order to change their self-efficacy beliefs. However, the fact that students have stronger beliefs than self-efficacy beliefs could be perceived as a positive result as they probably indicate that students realize the difficulties that they face and they do not tend to overestimate their abilities (Pajares & Miller, 1994). The accurate self-representation concerning the strengths and limitations is a presupposition for the development of self-regulation (Panaoura, Gagatsis, & Demetriou, 2009), an ability which is necessary in order to overcome difficulties. In fact, a priority of higher education should be to enable economics students to evaluate their own strengths and limitations and teach them self-regulatory strategies in order to persist in overcoming obstacles and difficulties during their studies. Their beliefs and their self-efficacy beliefs in the value of mathematics will be developed indirectly. This could also be true for science, engineering, and technology students, since mathematics is important for their studies as well.

The present study highlights the strong interrelations among students' beliefs and self-efficacy beliefs concerning the use of mathematics and representations effectively in understanding and presenting the concepts of the economic science. The university students' beliefs concerning the value of mathematics are of great importance. Hannula (2011) claimed that empirical research into mathematics-related beliefs indicates an overall pattern, where positive (or negative) beliefs are related to each other and to positive (or negative) emotions and positive (or negative) motivation. Believing that studying mathematics is important for their studies constitutes the first step in an attempt to make them believe that their abilities can change through hard work, persistence and insistence in facing and overcoming cognitive difficulties. Their positive strong beliefs concerning the value of mathematics for the economic science and the use of representations in mathematics could be reclaimed in order to overcome difficulties, develop self-regulatory strategies, improve their outcomes and consequently increase their self-efficacy. As Bartimote-Aufflick et al. (2015) indicate, university students' self-efficacy is higher under certain conditions than others and can be improved over time. For instance, research results indicate that self-efficacy was higher when particular teaching strategies were employed (Bartimote-Aufflick et al., 2015).

6. Conclusions and future study

The proposed structural model was found to function well in this study and revealed valuable information about undergraduate students' beliefs concerning the value of mathematics and representations for understanding functions, and their corresponding self-efficacy beliefs. However, the study was conducted at one site, with one questionnaire and one group of students. Furthermore, there is still information that needs to be discovered. It would be interesting in the future to examine the structure of students' beliefs and self-efficacy beliefs regarding the use of representations in relation to their achievement. Furthermore, it would be practically useful to investigate the impact of intervention programs that compare self-efficacy in a multi-representational learning environment for teaching mathematics courses in the economics science. Similar studies in other mathematics-related disciplines would also contribute to the discovery of students' needs in higher education.

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APPENDIX

Conceptualization of students' beliefs and self-efficacy beliefs

Affective factor	Questionnaire Item	Variables
Self-efficacy beliefs concerning Mathematics	I am able to understand mathematical concepts and procedures.	seBM1
	I feel confident when I have to use Mathematics	seBM2
	I am able to use Mathematics to solve financial/ management problems.	seBM3
	I have the ability to solve a financial/ administrational problem requiring the use of Mathematics.	seBM4
	I have the necessary skills to use and apply Mathematics.	seBM5
Self-efficacy beliefs concerning the use of representations	I am able to convert one representational system into another (e.g. from algebraic to graphical)	seBR1
	I am able to use representations effectively.	seBR3
	I have the ability to modify and use representations.	seBR4
	I can analyze the representations given to me.	seBR10
	I am able to recognize the graph of a function.	seBR11
	I can sketch the graph of a function.	seBR12
	I am able to apply properties for processing a representation.	seBR14
	I am able to use various forms of representation given to me effectively.	seBR15
Self-efficacy beliefs concerning the use of the representations for exponential functions	I have the ability to solve exponential functions.	seBRexp1
	I have the necessary knowledge about the representations of exponential functions.	seBRexp2
	I have the ability to use representations of exponential functions in my economic analysis.	seBRexp5
	I am able to use representations of exponential functions effectively.	seBRexp6
	I have the ability to use representations of exponential functions given to me.	seBRexp8
	I have the ability to analyze representations of exponential functions given to me.	seBRexp9
Self-efficacy beliefs concerning the use of representations for logarithmic functions	I have the ability to solve logarithmic functions.	seBRlog1
	I have the necessary knowledge of the representations of logarithmic functions.	seBRlog2
	I can convert the algebraic representation of the	seBRlog3

	logarithmic function into graphical representation.	
	I can convert the graphical representation of the logarithmic function into algebraic representation.	seBRlog4
	I prefer to solve problems that include representations of logarithmic functions.	seBRlog5
	I have the ability to use representations of logarithmic functions in my economic analysis.	seBRlog6
	I am able to use representations of logarithmic functions effectively.	seBRlog7
Beliefs concerning the use of representations	The graphical representation is a useful tool in problem solving.	BR1
	The graphical representation is an important way of understanding a concept.	BR2
	In order to solve some exercises, I have to construct a diagram.	BR3
	In order to solve many exercises, I have to understand the graphical representation given.	BR4
	Using representations can provide me with some useful information that might help me solve the corresponding problem.	BR5
	The construction of a representation to solve an exercise is helpful.	BR6
	The exercises in which graphical representation is given are easier to solve.	BR8
Beliefs concerning the importance of using representations in economics	The most appropriate way to present an economic analysis is through representation.	BiR2
	From the financial / administrative problems analyses some representations results which will need to understand and analyze were found.	BiR3
	Representations help me to solve financial / managerial problems.	BiR4
	In order to find the right solution to a financial problem, the use of representation is necessary.	BiR5
	Representations help me to understand a given financial situation.	BiR6
	The use of representations helps me to present financial concepts to the general public in a simple way.	BiR7
	The relationship between two variables is an exponential function is essential in many financial applications and analyses.	BiRexp3
	Mathematics theorems which deal with economic matters are good economic	BiM8

	analysis.	
Beliefs concerning the value of Mathematics for an economist	Mathematics is essential in my work.	BiM1
	The better I am at Mathematics, the better I will be in my work.	BiM3
	In our attempt to explain or analyze a dependent variable with respect to an independent variable, we use representations of exponential functions.	BiRexp4
	Representations of logarithmic function are an important tool for the students of economics / Finance / Administration.	BiRlog4
	The use of representations is helpful in my studies.	BiR1
Beliefs concerning the value of Mathematics for the science of economics	Without mathematics my science cannot exist.	BiM4
	Mathematical rules and procedures are necessary for financial/ administrative problems analyses.	BiM5
	Economic analyses which are emphasized in mathematical models correspond to reality.	BiM7
	Mathematics is used to explain the concepts of the economy.	BiM9
	The economic models are based on mathematical rules.	BiM10
Beliefs concerning the use of representations in the exponential functions	Representations of exponential functions are part of my everyday study.	BiRexp1
	Representations of exponential functions are very important in my science.	BiRexp2
Beliefs about the use of representations in the logarithmic functions	Representations of logarithmic functions are part of my daily study.	BiRlog1
	The representations of logarithmic functions are used in my science.	BiRlog2
	Economics /Finance / Administration students should be familiar with the use and processing of representations of logarithmic functions in their economic analyses.	BiRlog3