Multiple semiotic means in the use of formative assessment in secondary school mathematics

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Abstract. *The main idea of this contribution is to enhance the knowledge about the use of multiple semiotic means in formative assessment in mathematics teaching. Four teaching episodes of formative assessment situations in mathematics classrooms are discussed, focusing on the use and modifications of multiple semiotic means during formative feedback. Our findings reveal that multiple semiotic means such as gestures and different types of representations are involved during the process of feedback and different semiotic actions, such as treatments and conversions, take place, facilitating the interaction between teacher and students. Also, the type of feedback appears as a factor differentiating the type of semiotic means and semiotic actions.*

Keywords: formative assessment**,** feedback, semiotic means, treatments, conversions

1. Introduction

Recent international studies (e.g. OECD, 2012; Eurydice, Educational, Audiovisual, & Culture Executive Agency, 2012) have determined five main difficulties in the teaching and learning of mathematics. One of these difficulties refers to the improper use of formative assessment. The role and effectiveness of formative assessment has occupied several researchers of the mathematical community (e.g. Van De Walle, Karp, & Bay-Williams, 2013; Chappuis & Stiggins, 2002; Black & Wiliam, 1998). Previous research (e.g. OECD, 2005; Broadfoot, Weeden, & Winter, 2002) has highlighted the need for using particular teaching strategies in order to achieve an appropriate and effective use of formative assessment. Based on the role and the purpose of formative assessment (i.e. to improve students' learning and to rehabilitate their difficulties), it seems that this kind of assessment can be beneficial for both the students and the teacher. This is reinforced by the significant impact of feedback provided by students to teachers about what they know and what misconceptions they have (Hattie, 2009) in order to resolve any questions and difficulties about the concept that is taught.

Arzarello, Paola, Robutti, and Sabena (2009) suggest that during the process of mathematics teaching and learning, a diversity of actions is produced both by the students and by the teacher through different semiotic sources. Thus, semiotic means can have an essential role in the formative

assessment process also and mainly during the provision of feedback, as different kinds of semiotic systems can co-exist during this process. For instance, gestures, glances, drawings and extra-linguistic means of expression seem to be key components of semiotic activities carried out in class.

In relation to the above, our examination focuses on the contribution of multiple semiotic means in the use of formative assessment in mathematics teaching. More specifically, this study focuses on providing formative feedback during formative assessment and the role of different semiotic means in this process. In particular, we aim to answer the following questions:

1. Which semiotic means are involved during the process of providing formative feedback?
2. What are the interactions and relations between these different semiotic means during formative assessment?

We will try to approach these questions by analyzing teaching episodes from mathematics classrooms, in which the teacher try to engage the students in formative assessment processes, using different techniques and semiotic means for this purpose.

2. Theoretical background

2.1. Procesos de idealización y de materialización

Many definitions have been provided about formative assessment. Recent definitions about this kind of assessment describe formative assessment as a way of assessment which checks who is learning or not and helps teachers design their next lesson (Van De Walle, Karp, & Bay-Williams, 2013). More specifically, formative assessment is defined as "a process where the teachers gather information about their students' learning and the teaching is modified as a result of the feedback that they provide to their students" (Cauley & McMillan, 2010). The National Council of Teachers of English stress that the results of formative assessment help teachers to make decisions due to the fact that this type of assessment provides information about the difficulties of the students, their misunderstandings in a concept, their strengths and their weaknesses and leads teachers to change their teaching techniques in order to achieve their teaching goals (NCTE, 2010).

Previous definitions about the formative assessment refer to the "assessment that is specifically intended to provide feedback on performance for improving and accelerating learning" (Sadler, 1998). Black and Wiliam (1998) highlight the active role of students in this process, supporting that formative assessment makes students responsible for their learning because they can assess their work, draw conclusions about their learning and plan next steps for further progress. In accordance to this, Harlen (2000) points out that "children have a role in assessment for this purpose since it is, after all, the

children who do the learning" (p. 112). That is why many researchers stress that assessment must be formed "for" learning and not "of" learning, as it is generally acknowledged that increased use of formative assessment (or assessment for learning) leads to higher quality learning (Wiliam, Lee, Harrison, & Black, 2004).

A definition that combines many key points about formative assessment is the one proposed by Popham (2008, p. 5), which is accepted by the Formative Assessment for Teachers and Students (FAST) group as the most accessible to educators (Clark, 2011b). According to this definition, formative assessment is "a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes".

2.2. Techniques for formative assessment

Assessment techniques and their effects on the students' learning, but also their emotional aspect have attracted the interest of several researchers (i.e. Crooks, 1988; Black & Wiliam, 1998) in the last decades and many of them suggest different formative assessment techniques, many of which present common points. For example, Cauley and McMillan (2010) report that informal observations and oral questions posed to the students concerning the content being taught or reviewed are two techniques that allow continuous formative assessment. And when the information from the observations and the questions posed to students is accurate, the teacher identifies the instructional adjustments that can help students improve their learning. Cauley and McMillan (2010) emphasize also the technique of providing clear learning goals to the students, explaining that formative assessment is more effective when the students have a clear idea of their teachers' expectations, because giving clear expectations allow students to set realistic and achievable goals.

Clark (2010) provides a richer list of sixteen teaching formative assessment techniques that involve the students in a reflective thinking and problem solving, considering the technique of questions as the most significant one. Feedback provided to students as comments and not as grades, oral feedback to the students, sharing the assessment's criteria with students, peer-assessment and setting learning goals with the students are some of the formative assessment techniques he proposed.

The techniques proposed by Clark (2010) are identified in other researchers too. For example, the General Teaching Council for England (GTCE, 2011) considers the effective use of questioning techniques, the use of marking and feedback strategies, the sharing of learning goals to students and peer and self-assessment by pupils to be key characteristics of formative assessment. Therefore, it seems that there is a general agreement among the various researchers regarding the techniques that are considered important for

the effective implementation of formative assessment in teaching of mathematics.

2.3. The effective use of formative feedback

As previously mentioned, in this contribution we emphasize on feedback provided during the formative assessment process in the mathematics classrooms we have observed. Feedback emerges an important dimension of formative assessment, as several definitions of formative assessment highlight the importance of its integration in the teaching. For example, Nicol and Macfarlane-Dick (2004) argue that assessment can be characterized as formative when it generates information for feedback that can be used by students to enhance their learning and their success. This information is useful to teachers in order to adapt their teaching to the needs of their students.

According to different references on feedback, three types of feedback are discriminated. The first type refers to feedback provided by the teacher to the students aiming to help them overcome their difficulties and improve their performance in the particular content they are taught. In this sense, Sadler (1998) refers to formative assessment as a process that seeks to provide feedback on the performance of students in order to improve and accelerate their learning. The second type of feedback refers to the information given by the students to the teacher in order to help him/her to decide how to modify the teaching process for helping students to strengthen their understanding. Actually, Hattie (2009) stresses that a strong influence of formative assessment on the students' performance is achieved by the substantial feedback provided by the students to the teacher regarding their understanding, their mistakes or misconceptions. Furthermore, feedback can be provided not only by the teacher to the students and vice versa, but often peers can provide feedback to each other. For example, in the context of a group work, the students provide their own feedback to their peers, while they cooperate for carrying out the work (Nicol & Macfarlane-Dick, 2004). Therefore, any direction formative feedback gets, it benefits both the students and the teachers.

However, feedback is not always formative, as there are certain factors that determine when it is formative or not. In particular, feedback becomes formative when students are a) involved in a process, which focuses on meta-cognitive strategies, b) are supported in their efforts to reflect upon for their work, c) understand the link between their previous performance, their current understanding and the clearly defined success criteria, and d) are active as responsible of their own learning (Clark, 2011a).

2.4. The use of semiotic means in the teacher-student interaction

In a semiotic approach to mathematical teaching, the role of signs and the way they are adopted by individuals within their social context is central

(Arzarello, Ferrara, Paola, & Robutti, 2005). The term "semiotic" means "theory of signs" (Nöth, 2000). According to Peirce, a sign is anything that "stands to somebody for something in some respect or capacity" (Peirce, 1931-1958). Saussure (1959) defined the "sign" as a combination of two mental constructs: a "signified" together with its "signifier". Sometimes a "signifier" can be arbitrary, as it is related to the "signified" with a social condition (an agreement, a rule). In this case, the observer cannot discover himself the "signified" through the "signifier", as he has to be aware of this social condition. Such kind of optic "signifiers" include the written language, the traffic lights, gestures etc.

Arzarello, Paola, Robutti, and Sabena (2009) claim that during the learning processes in the mathematics class, a variety of semiotic actions and productions are activated by the students and by the teacher using different resources: words (orally or in written form), extra-linguistic modes of expression (gestures, glances), different types of inscriptions (drawings, sketches, graphs), various instruments (from the pencil to the most sophisticated information and communication technology devices) and so on. Such resources can be used with great flexibility, as the same person can exploit many of them simultaneously. Sometimes, they are shared by the students (and possibly by the teacher) and used as communication or thinking tools, with the actions and productions they support to be important for grasping mathematical ideas. In fact, all such resources help to bridge the gap between the worldly experience and the more formal mathematics.

Within this wide perspective, Arzarello (2006) has introduced the "semiotic bundle", which allows studying gestures – and teaching-learning processes – in a multimodal approach. A "semiotic bundle" is a system of signs – with Peirce's comprehensive notion of sign – that is produced by one or more interacting subjects and that evolves in time. Typically, a "semiotic bundle" is made of the signs produced by a student or by a group of students while solving a problem and/or discussing a mathematical question. An important example of "semiotic bundle" is given by the unity speech-gesture. McNeill claimed that gesture and spoken utterance should be regarded as different sides of a single underlying mental process (McNeill, 1992). Gesture and language constitute a "semiotic bundle", made of two deeply intertwined semiotic sets. Researches on gestures have discovered some important relationships between the two, for example match and mismatch has been studied (Goldin-Meadow, 2003).

Thus, an essential aspect of the analysis of gestures is the relationship between the content of gestures and speech. On the one hand, gestures may convey the same information as speech (Arzarello & Edwards 2005), thus reinforcing the speech meaning (Göksun, Hirsh-Paseka, & Golinkoff, 2010). On the other hand, gestures and speech may contain different information. Gestures may provide information that is conflicting to the content of speech,

or they may supplement speech by providing additional information. Such a speech–gesture mismatch is seen as an indication for a transitional stage in cognitive development or in mastering a task (Alibali, Kita, & Young, 2000; Goldin-Meadow, 2003).

Elia and Gagatsis (2016) relate Kaputs' definition of representations to the concept of gestures, showing that (iconic) gestures can be considered as semiotic means, as they are in line with this definition. The definition of representations in relation to gestures is the following:

1. The entity which is represented (e.g. two-dimensional geometrical figures)
2. The entity which represents (e.g. diagrams of geometrical figures and gestures)
3. Elements of the entity which is represented (e.g. directions of line segments, vertical lines, parallel lines, intersecting sides etc.)
4. Elements of entity which represents (e.g. gestures representing parallel or vertical lines etc.).
5. Correspondence between 3 and 4 (parallel or vertical lines with the relevant gestures).

Furthermore, based to the previous definition, gesture satisfies also the discrimination of representations to internal and external. In fact, gestures are considered as external representation in close interaction to internal representations. Through gestures we are able to display orally our internal thoughts and the way we understand the world. Thus, gestures can be taken as the "windows" of our internal thoughts or as "pipes" transmitting ideas that are already in our minds and wait for the proper material or verbal expression. In fact, McNeill (1992) proposed four categories of gestures with respect to their meaning: (1) deictic gestures, pointing movements to existing or virtual objects and actions in space; (2) iconic gestures which are closely related to the semantic content of speech, that is, they visually represent the content of concrete entities and actions, (3) metaphoric gestures, which represent an image of an abstract object or idea; (4) temporal highlighting gestures, simple repeated gestures used for emphasis. For our analysis further on, we use this framework for identifying the type of gestures that appear in the teaching episodes we examine and discuss their function during providing feedback.

Besides students, the teacher participates also in this semiotic production, and thus the "semiotic bundle" may include also the signs produced by the teacher (Arzarello, Paola, Robutti, & Sabena, 2009). Furthermore, the process developed by the teacher using semiotic sources in order to strengthen the construction of knowledge is called "semiotic game" (Arzarello & Robutti, 2008; Arzarello & Paola, 2007). In particular, a "semiotic game" takes place when the teacher responds to the semiotic resources that the students produce and then he/she directs the construction of knowledge taking into account these sources (Arzarello, 2006). In fact, the most important mathematical use

of semiotic means is their internal potential to be changed into other semiotic means. In order to analyze the cognitive processes underlying any mathematical activity, and problems of students' comprehension in learning mathematics we must carefully distinguish these two types of semiotic change (Duval, 2008). The first produces a semiotic mean of the same type as the starting representation. On the other hand, the second produces a semiotic mean of a different type. They are respectively called treatments and conversions.

In studying a semiotic game between a teacher and a kindergarten student, Elia, Gagatsis and Van den Heuvel-Panhuizen (2004) conducted a case study which explores the function of gestures in a geometrical activity at kindergarten level. In their study, the spontaneous gestures of the child were investigated, as well as the influence of the teacher's gestures on the child's gestures. Actually, the major goal of the study was to unravel the role of gestures in using and communicating spatial and shape related ideas by a kindergarten child that was engaged in an activity requiring the transformation of spatial constructions into verbal descriptions. Their results showed that the child was spontaneously using iconic and deictic gestures throughout the whole activity. These gestures, and primarily the iconic ones, helped her make apparent different space and shape aspects of the constructions she was making. Along with her speech, gestures acted as semiotic means of objectification to successfully accomplish the task. The teacher's gestures were found to influence the child's gestures when describing aspects of shapes and spatial relationships between shapes, as the child was either mimicking or extending the teacher's gestures. Watching and mimicking or extending the teacher's gesture and speech acts helped the child enter into a process of objectification (Radford, Bardini, & Sabena, 2007) for these concepts.

3. Methodology

For answering our research questions, the data collection was conducted through videotaping two consecutive mathematics lessons at lower secondary school, emphasizing at moments of formative assessment. This data collection was included in the actions of a European research project about the use of formative assessment. The Formative Assessment in Mathematics Teaching and Learning (FAMT&L[1]) is a European research project which aims to design a virtual environment (a web repository) for in-service and pre-service teachers' training about the proper use of formative assessment in teaching-learning situations and in elaborating a training model (or methodology) for mathematics teachers training in secondary school. The purpose of this

[1] [538971-LLP-1-2013-1-IT-COMENIUS-CMP]

training model (or methodology) is twofold: to improve teachers' competences a) on educational planning and assessment (both formative and summative assessment; assessment for learning) and b) on mathematics didactics.

This project begins from an investigation of the mathematics teachers' beliefs about formative assessment (Michael-Chrysanthou, Lovece, Vannini, & Gagatsis, 2016; Lovece, Vannini, Michael-Chrysanthou, & Gagatsis, 2016; Michael-Chrysanthou, & Gagatsis, 2015; Michael-Chrysanthou, Gagatsis, & Vannini, 2014) in order to develop model (using e-learning) for middle school math teachers (that can be applied to in-service and pre-service training) and then to conclude a design of an appropriate virtual environment for in-service and pre-service teachers. This learning environment will provide a variety of tools and objects (examples of learning contexts, video of situations of teaching mathematics, assessment tools, training paths and their specific use in the teaching of mathematics), including a guideline to be used in in-service secondary schools teachers training courses. The FAMT&L project is carried out by five partner EU universities: The Alma Mater Studiorum Università di Bologna – Departments of Education and Mathematics, which is the Project Coordinator, the University of Cyprus – Department of Education, the University of Applied Sciences and Arts of Southern Switzerland – Department of Formation and Learning, the Cergy-Pontoise University – University Institute of Teachers Training and the Inholland University of Applied Sciences.

For the purposes of this contribution four teaching episodes from a grade 7 mathematic classroom were analyzed. These episodes were extracted from two consecutive lessons about "*Integers-Rational numbers*" and in particular about the "*Multiplication of Rational number*" and the *"Inverse numbers"*. The learning goals of the lessons were students to be able: a) to estimate the product of numbers with same and different sign, and b) to estimate and find the inverse number of each rational number. In the episodes, the participants are involved in formative assessment situations. In these lessons, the teacher applies the "traffic lights" technique for engaging students in the process of providing them feedback.

Actually, the students use three cards in different colors (green, orange and red) for giving feedback to their teacher regarding their understanding about the content that is taught. The green card represents a good understanding giving the teacher a "green light" to continue the teaching process. The orange card indicates that the students have a question/misunderstanding and they need for additional help. The red card is used by the students that have a poor understanding of the mathematical concept, so that the teacher to explain it again. As each of the cards has its own meaning, we consider the "traffic lights" technique as a semiotic source and the cards as an arbitrary sign, as defined by Saussure. In fact, the traffic lights cards can be taken as an arbitrary signifier, which is related to its signified through a social condition

(agreement – rule). Thus, an observant cannot discover himself the signified through the signifier, as it is necessary to be aware of this condition. Therefore, the traffic lights cards, as well as gestures and written language are considered as visual signifiers. Apart from these semiotic means, the use of formative feedback is also carried out in an informal way, through the teachers' oral questions during teaching and students' questions without the use of their cards. These ways are considered to promote the use of formative feedback and the interaction between the teacher and the students and between peers.

The analysis of these didactic episodes is based on Arzarello's and his colleagues (2009) approach of the synchronic analysis and the diachronic analysis. Synchronic analysis enables us to concentrate on the interrelations between different semiotic resources, including gestures and oral language, activated by the subjects (i.e. students and teacher) simultaneously at a specific moment. By implementing diachronic analysis we can identify changes in the way the participants use gestures in relation to verbal representations and other semiotic resources, in successive moments over a short period of time, that is, within a mathematical activity or lessons, over a medium period of time, that is, between consecutive lessons, and over a long period of time, that is, between the different periods the observations will take place within the school year. We consider this approach suitable for analyzing our teaching episodes, as we are interested in tracing the semiotic means (gestures, oral and written language, traffic light cards) involved during the process of formative assessment and the interactions between them, but also in tracking changes in the way these semiotic means are used by the teacher in the two successive lesson we observed.

4. Results

Lesson 1 – Episode 1

In the first episode we first observe a teacher – student interaction, at first during the teachers' effort to provide individualized feedback to a student that appears having difficulties. Then, the teacher opens the discussion to the whole class posing oral questions, in order to engage the rest of the students in the process of providing feedback to the student having difficulties. So, there is an indirect interaction between the students during peer-feedback with the mediation of the teacher.

In particular, during the teacher's discussion with Student 1 (S1), she traced that S1 faced difficulties in multiplying $8\times(-1)/(8)$. In fact, S1 indicated difficulties in realizing that an integer number could be represented as a fraction. So the teacher called him to recall the rule for the multiplication between fractions. In order to help him recall the rule, the teacher used a deictic gesture (Figure 1a) to show a previous example written on the board,

for turning John's attention to its solution. Using this deictic gesture that corresponded to her verbal expression, she gave feedback to the student by helping him realizing that the same kind of operations is needed for the previous example and this new example.

After the teacher realized that the particular student needed further feedback for understanding the proper procedure, she tried to apply the peer-feedback technique by posing oral questions for encouraging the rest of the students to help their peer. In helping S1 by explaining the multiplication procedure, Student 2 (S2) suggested they should use a "deletion", referring to simplification. She was, probably, unable to use the proper term, as the term of simplification was new for the students at this moment. However, in the context of a semiotic game, the teacher made a treatment to S2's verbal expression and gave the right mathematical term "simplification". Then, the teacher encouraged students to think how to convert number 8 to a fraction in order to be able to execute the operation. Another student using an informal mathematical language suggested the teacher to "draw a line and below it to write number one". The student used "line" referring to fractional line and the word "below" for the denominator. Then, the teacher converted on the board these verbal expressions into the symbolic representation "8/1".

The same student continued by explaining verbally the procedure of multiplying the two fractions using simplification. At the same time, the teacher was converting S2's verbal explanations into a written symbolic expression on the board, to highlight each step of this procedure. In order to help S2 find the sign of her final answer, the teacher repeated verbally the procedure of multiplication, performing deictic gestures at the same time for pointing at each number or sign she was referring to.

Coming back to S1, the teacher tried to provide him once again individualized feedback, by trying to give him particular advice in order to correct his mistakes. She actually advised him to be more careful at the position of the numbers, when writing a fraction next to an integer. During this effort, the teacher synchronized her verbal explanations to a symbolic form on the board, for visualizing the proper way of representing these numbers. For further stressing this proper way of writing, the teacher then used once again deictic gestures (Figure 1b) synchronized to her speech, for pointing at the right position of each digit of the numbers as appeared in a previous example on the board.

In the semiotic game during the teachers' feedback to students in this episode, conversions of verbal to written symbolic expressions in combination to deictic gestures were evident. Thus, we consider that the teacher used these conversions of representations as a formative assessment tool, for providing feedback to them.

(a)

(b)

Figure 1. Deictic gesture of teacher about the previous example (a) and deictic gesture of teacher about the proper position of numbers (b).

Lesson 1 – Episode 2

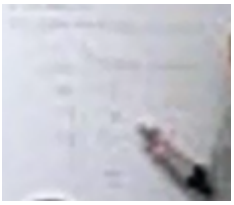
In this episode the teacher asks the students to provide her feedback using their traffic lights cards (Figure 2a) about their understanding regarding the inverse and opposite numbers. Based on the students' reactions using their cards, a teacher-student interaction during the teachers provides feedback to a particular student having difficulties is analyzed.

In particular, the teacher observed a few students displaying an orange card about their understanding. For providing them feedback, she asked them to express their questions about the mathematical content that was still fuzzy to them. A student expressed his difficulties in understanding why the inverse number of $1/3$ is 3. Then, the teacher encouraged him to revise the previous written examples on the board, by posing oral questions to him in order to guide his recall the right procedure. Actually, the teacher asked the student to explain her how the fraction $1/2$ was transformed to the integer 2, by using deictic gestures at the same time for pointing at each number she referred to (Figure 2b). After the student answered that 2 is actually $2/1$, the teacher translated this verbal expression to a symbolic expression on the board by drawing a fraction line and writing the denominator 1 below it. The same procedure followed the teacher's question about what actually number 3 is, by using also a deictic gesture for each number. The student replied that the number 3 is equal to $3/1$ and the teacher converted this answer by writing it on the board. Then, the teacher asked the student about the fraction $1/3$, by using also a deictic gesture for pointing it on the board. In relation to his previous answer, at this moment the student realized that the inverse number of $1/3$ is 3.

Thus, in this episode the teacher tried to help the student overcome his difficulties by recalling previous examples written on the board. Her oral questions, the conversion of the students' oral expression to written symbolic representation on the board and the synchronized form gestures with her verbal expressions were the semiotic means used by the teacher in the semiotic game held in this episode. Therefore, the use of traffic light cards as a semiotic means was a source of feedback for the teacher, who then used a semiotic bundle comprised of oral language, symbolic representations and gestures for providing feedback back to the students. As a result, the interrelation between

the different semiotic means observed in this episode was decisive for the teacher – student interaction during providing feedback to each other.

(a)



(b)

Figure 2. Traffic light cards for the children understanding about inverse and opposite numbers (a) and teacher's deictic gesture about the numerator and denominator of $1/2$ (b).

Lesson 2 – Episode 1

In this episode two phases of teacher-students interaction while providing feedback take place. At the beginning of the episode the teacher asked the students to express any questions they might have about the lesson. She observed no reaction from the students and in keeping trying to gain feedback from them, she urged the students to use their traffic light cards for expressing their understanding about the lesson. The students reacted and raised their cards, so the teacher was able to trace students that still have doubts about the mathematical content of the lesson. These were the students that raised an orange card (Figure 3a). So, the teacher focused on these students and started posing them questions for finding their exact difficulties and being able to provide feedback to them. It is important to stress that, in this case, the use of a semiotic means (the traffic light cards) facilitated the students-teacher interaction, as it enabled the students express the degree of their understanding. This is probably attributed to the fact that when the students who have a question they feel uncomfortable to ask for help in the case the rest of the students don't have any question.

In the first phase of teacher-student interaction during the feedback process, the teacher asked a student with an orange card to describe her difficulties. This student faced difficulties while converting a decimal number into an improper fraction for finding next the inverse number (the student explained that she hasn't understood very well why 2.5 was turned into a fraction). The teacher, instead of answering herself, she transmitted the question to the rest of the students, in order to engage them in a peer-feedback procedure. Then, a second student answered the question, by trying to explain the right procedure. During this students' verbal explanation, the teacher contributed to this semiotic game by making treatments and conversions of the semiotic means produced by the particular student. In fact, the teacher made a treatment of the second student's verbal explanation, by rephrasing her answer

and by adding clarifications in order to describe the rule for inverting fractions in a more clear way. After the explanation of the rule, the first student expressed another question she had. After that, the teacher provided feedback directly to the student, by using a deictic gesture to point at a fraction, in order to repeat and stress the rule she mentioned previously ("*We invert only fractions and not decimals*"). Thus, in this phase the teacher at first provides indirect feedback to the students (by enabling other students in providing peer-feedback) through treatments of semiotic means and then provides direct feedback to the students by involving other semiotic means, such as gestures.

At the second phase of teacher-student interaction during feedback, the teacher focused on a second student that expressed his difficulties by raising an orange traffic light card. Specifically, this student had difficulty regarding the inverse number of $-1\ 2/7$. The teacher in order to start providing him feedback, she posed oral questions for guiding him towards the right procedure. At the beginning, she asked the student which is the inverse number of $-1\ 2/7$ and the student replied with a wrong answer ($-1\ 7/2$). The student has actually inverted only the fractional part of the mixed number, without transforming the mixed number to an improper fraction. Then, the teacher made a metaphoric gesture of a circle around the number $-1\ 2/7$, which was previously written on the board, in order to remind and stress the rule that "*we invert the whole mixed number*". In continuing stressing the rule, while explaining that "*for finding the inverse number we inverse the whole fraction*", she used an iconic gesture for showing the action of inverting the denominator with the numerator. A deictic gesture followed in order the teacher to indicate the fractional part of the mixed number ("*You have only inverted the fractional part…"*) and then a temporal highlighting gesture was used for showing the whole mixed number ("…*of the mixed number*"). In this gesture the teacher made a repeated motion of showing each part of the mixed number (the integer and then the fraction) consecutively, in order to help the student realize that these two parts are related and constitute a mixed number. Next, the teacher explains that "*By inverting* $\frac{2}{7}$ *to* $\frac{7}{2}$ *you don't invert the rational number"*), while at the same time she uses a deictic gesture pointing at the denominator (7) and numerator (2) respectively and again a metaphoric gesture of a circle around the number $-1\ 2/7$ in order to show the rational number. Thus, the teacher's blended character gestures were synchronized with her verbal expression.

After providing feedback to the student by explaining the rule, by involving different gestures, the teacher asked the student to explain her again the right procedure. The student answered correctly that "*we first have to turn the mixed number into an improper fraction*" and continued orally to the solution of the task, which was translated to a symbolic representation on the board by the teacher. Therefore, the teacher's semiotic game while providing

feedback was found effective, as the student realized his mistake and followed the right procedure.

(a)

(b)

Figure 3. Use of traffic cards about the inverse numbers (a) and metaphoric gesture of a circle around the number $-1\ 2/7$ (b).

Lesson 2 – Episode 2

In this episode also, using the traffic lights technique, the teacher encouraged the students to express their difficulties about the rational numbers. In this case many orange cards were observed and less red cards. Based on the students' instant feedback using a semiotic means for expressing their degree of understanding, the teacher decided to modify her teaching and pay more attention at the points the students were still facing difficulties. The students expressed a fuzzy understanding about the inverse numbers of mixed fractions and the conversion of decimals to fractions. Thus, the teacher decided to give some extra tasks on the board and provided students time to solve them individually. After the students finished with the solution of the tasks, the teacher opened the discussion of these tasks, by asking particular students to explain their solutions. The teacher focused mainly on the students that have previously displayed an orange or a red traffic light card. Therefore, in this episode an interaction between the teacher and many students takes place.

At the beginning of the discussion of the solutions with the students, the teacher asked the students to explain their solution. During the students' oral explanations, the teacher was converting them into symbolic representations on the board and was also making treatments of, by expressing orally the meaning in a more correct and completes form. In order to check the effectiveness of feedback the students have received through the discussion of the first two tasks, the teacher asked them once more to express their understanding using their traffic lights cards. From the students' feedback, the teacher was able to trace that the students had still difficulties on how to turn a mixed number into an improper fraction.

In fact, a student mentioned that she has not understood very well the previous example about the inversion of 2.72. For specifying the student's question, the teachers asked "*Do you mean how we have turned the mixed number into an improper fraction?*". As in the previous episode, while asking she used a metaphoric gesture of a circle around the mixed number and a

deictic gesture for pointing at the improper fraction. She then repeated the question for the whole class by using a deictic gesture while mentioning the mixed number and the improper fraction respectively. Thus, in this case also the teacher produced a semiotic bundle, by matching speech and gestures.

Next, the student started explaining to the teacher the procedure she followed for turning 2.72 into an improper fraction. The teacher asked her to solve the following example (inversion of 3.136) for checking her understanding. In trying to help her focus on the number of decimal digits, she made treatments of a previous student's oral answer by paraphrasing it ("*Ann told us that*…"). In addition, the teacher synchronized speech and gestures, using a deictic gesture pointing at the digits of the decimal number (2.72) of the previous example for helping the student remember that "*when the digits are two the denominator of the fraction is 100*" and pointing also at each number of the fraction 72/100. The teacher then repeated the deictic gesture for the decimal digits of number 3.136 for helping the student answer that "*when the digits are three the denominator of the fraction is 1000*". After, this students-teacher interaction the student proceeded to the verbal explanation of the solution of the task and the teacher to the conversion of the explanation into a symbolic form on the board. This semiotic game seemed effective in the following minutes of the episode, as the students provided right solutions for inversions including decimal numbers.

5. Discussion

The analysis of the teaching episodes presented above aimed at first in tracing the semiotic means which are used either by the teacher or the students during formative assessment and mainly while providing formative feedback, from the teacher to the students or between students with the guidance of the teacher. For this purpose Arzarello's and his colleagues' (2009) synchronic analysis was used, which allowed us to identify a diversity of semiotic means involved in the formative assessment process. Although all the teaching episodes we discuss were carried out during the teaching of Algebra content, mainly related to symbolic representations, our results showed that other semiotic means can have an essential role during this process. In fact, through the analysis of the four teaching episodes, the teacher was observed to mainly use oral language, written symbolic expressions and gestures. Thus, the teacher was producing semiotic bundles comprising of a multimodality of semiotic means. As stressed by Arzarello and Edwards (2005), multimodality consists in interactions among the different registers within a unique integrated system, composed by different modalities: gestures, oral and written language, symbols, and so on.

As for the students, besides oral language, they had the chance to use another semiotic means for providing instant feedback to their teacher about

their level of understanding about the content of the lesson. The use of traffic lights cards as a semiotic means gave the students the flexibility to express their questions at any time they felt they haven't understand the mathematical content very well. The particular technique helped also the teacher identify the students' difficulties instantly and provide immediate feedback to them according to their needs. Therefore, the use of traffic lights cards as a semiotic means can facilitate the interaction between all participants (teacher and students) during the formative assessment process. With this semiotic means the teacher receives feedback about the effectiveness of teaching and the students' understanding and this helps the teacher decide how to modify the next steps in order to help students face their difficulties. Thus, this semiotic means creates interplay between the students and the teacher, which facilitates the semiotic game between them.

Besides identifying the type of semiotic means that were involved in our teaching episodes, we are also focused on examining how these different semiotic means were related in order to contribute to the production of semiotic bundles and to the semiotic game between the teacher and the students. Our results revealed that the semiotic means identified in the process of gaining or providing feedback were related between them, as conversions from one semiotic means to another were observed. In fact, conversions from verbal expressions to symbolic representations were often conducted by the teacher, during trying to provide feedback to the students in relation to the difficulties they have expressed. In addition the teacher was frequently producing gestures that were synchronized to her verbal expression; thus it can be consider as another type of conversion, from verbal expressions directly to gestures. In fact, the synchronization of speech and gesture is considered by Sfard (2009) to increase the effectiveness of teaching actions. Besides conversions, the teacher was observed to make treatments of the semiotic means used by the students. This mainly regards treatments of the students' oral productions, as many times the teacher was using the students' answers for repeating them in order to stress something or for expressing the meaning in a more complete form, using a more proper mathematical language.

It is, thus, evident that the transformation of semiotic means, either by treatments or conversions, is an important process for constructing mathematical knowledge and communication during the semiotic game between the teacher and the students. In agreement to this, Duval (2008) stresses that mathematics activities require the possibility of using various semiotic means and intrinsically consist in the transformation of semiotic means (Duval, 2006). It is, therefore, necessary teachers to get aware of the necessity of such kind of processes and actions during their teaching, besides the importance of incorporating multiple semiotic means.

In extending the analysis of our teaching episodes, the diachronic analysis of Arzarello and his colleagues (2009) was also considered necessary for

helping us form a more complete idea about the teachers' semiotic actions during providing feedback to the students. Indeed, the diachronic analysis allowed us to identify changes in the teachers' semiotic productions. In all the episodes the teacher was mainly producing oral questions and making treatments of the students' verbal expressions and conversions from verbal to symbolic expressions and from verbal expressions to gestures. What differentiates in the teacher' semiotic productions are the kind of gestures, which get richer from the first lesson to the second. In particular, in the two episodes of lesson 1 the teacher produces only deictic gestures, synchronized to her verbal expressions. In the next two episodes from lesson 2 the teacher is observed to produce a diversity of gestures, as besides deictic gestures in the first episode, metaphoric and temporal highlighting gestures are also present. Thus, we observe a change in the teachers' kind of gestures with the same lesson (lesson 2) and from lesson 1 to lesson 2. In fact, Alibali et al. (2000) consider that gestures affect the mental representations of the speakers focusing their attention on specific features of the particular case.

Looking deeper at the 1st episode of lesson 2, it is interesting to focus on the changes traced in the teachers' semiotic productions between the two phases within this episode, in relation to the type of feedback. Actually, in the first phase of the episode the teacher involved the students in a peer-feedback process. Instead of providing feedback herself, the teacher guided the students in this process mainly by making treatments of the students' verbal expressions and by posing additional oral questions. She, then, provided direct feedback to the students by producing a semiotic bundle of deictic gestures and speech. Later on, at the second phase of the same episode, a teacher-student direct interaction took place, in which the teacher's semiotic bundle consisted of gestures with a blended character, as additional kinds of gestures were apparent.

Thus, at a first level, the teachers' semiotic means were differentiated in relation to the type of feedback, either indirect through guiding peer-feedback or by direct feedback to the students. At a second level, the differentiation regarded the semiotic means involved in the process of providing direct feedback to students, as at the beginning the teacher used a more simple type of semiotic bundle (speech and deictic gesture), whereas further on the semiotic bundle gained a more complex form (speech and different types of gestures). Consequently, we can say that the type of semiotic productions can be affected by the type of feedback (direct or indirect), but also within the same type of feedback different types of semiotic means are possible to be present. Therefore, the teachers should reflect about which are the proper semiotic means according to the type of formative assessment situations they are creating and according to the necessary modifications of their teaching actions based on the students' needs.

Concluding, our analysis highlighted the importance of using multiple semiotic means and transformation actions, such as treatments and conversions, in facilitating the interaction between the teacher and the students when applying formative assessment. Consequently, the use formative assessment actions in relation to the use and transformation of multiple semiotic means can contribute to the development of the students' cognitive, but also social structures, as learning should promote interaction and positive interdependence among students (Johnson & Johnson, 1996).

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